

Selections from S. M. Henderson, R. L. Perry, and J. H. Young.
1997. Principles of Process Engineering, 4th ed. ASAE.

Table 2-1. Viscosity of various materials

	Temp. (C)	Mass Density (Approx. kg/m ³)	Dynamic Viscosity (Pa s)	Kinematic Viscosity (m ² /s)
Air	0.0	1.295	1.71×10^{-5}	1.33×10^{-5}
	21.0	1.202	1.82×10^{-5}	1.51×10^{-5}
	100.0	0.947	2.19×10^{-5}	2.30×10^{-5}
CaCl ₂ brine, 24% sol.	-23.0	1238.0	1.25×10^{-2}	1.01×10^{-5}
	-18.0	1234.0	8.80×10^{-3}	7.13×10^{-6}
	2.0	1227.0	3.69×10^{-3}	3.01×10^{-6}
Cotton-seed oil	16.0	920.0	9.10×10^{-2}	9.89×10^{-5}
Cream, 20% fat, past. 30% fat, past.	3.0	1010.0	6.19×10^{-3}	6.13×10^{-6}
	3.0	1000.0	1.39×10^{-2}	1.39×10^{-5}
Freon-12	-15.0	1440.0	3.30×10^{-4}	2.29×10^{-7}
Liquid Ammonia	-15.0	660.0	2.50×10^{-4}	3.79×10^{-7}
	27.0	660.0	2.10×10^{-4}	3.50×10^{-7}
Lub. oil, SAE 10 SAE 30	16.0	900.0	1.00×10^{-1}	1.11×10^{-4}
	66.0	870.0	1.00×10^{-2}	1.15×10^{-5}
	16.0	900.0	4.00×10^{-1}	4.44×10^{-4}
	66.0	870.0	2.69×10^{-2}	3.09×10^{-5}
Milk, skim whole	25.0	1040.0	1.37×10^{-3}	1.32×10^{-6}
	0.0	1035.0	4.29×10^{-3}	4.14×10^{-6}
	20.2	1030.0	2.13×10^{-3}	2.07×10^{-6}
Molasses, heavy dark	21.0	1430.0	6.59	4.61×10^{-3}
	38.0	1380.0	1.88	1.36×10^{-3}
	49.0	1310.0	9.20×10^{-1}	7.02×10^{-4}
	66.0	1160.0	3.74×10^{-1}	3.22×10^{-4}
NaCl brine, 22% sol.	-18.0	1190.0	6.10×10^{-3}	5.13×10^{-6}
	2.0	1170.0	2.69×10^{-3}	2.30×10^{-6}
Soybean oil	30.0	920.0	4.06×10^{-2}	4.41×10^{-5}
Sucrose, 20% sol. 60% sol.	0.0	1086.0	3.81×10^{-3}	3.51×10^{-6}
	21.0	1082.0	1.92×10^{-3}	1.77×10^{-6}
	80.0	1055.0	5.92×10^{-4}	5.61×10^{-7}
	21.0	1289.0	6.01×10^{-2}	4.66×10^{-5}
	80.0	1252.0	5.42×10^{-3}	4.33×10^{-6}
Water	0.0	1000.0	1.80×10^{-3}	1.80×10^{-6}
	21.0	998.0	9.84×10^{-4}	9.86×10^{-7}
	49.0	987.0	5.58×10^{-4}	5.65×10^{-7}

2.14 Energy Loss Due to Flow Through Agricultural Products

The resistance of fixed beds of granular materials to air flow is a function of the size and shape, surface configuration, and size distribution of the granular particles and to the method of pouring the material into the container which affects the amount of void space in the bed.

Zenz and Othmer (1960) and Leva (1959) have reviewed a large number of carefully conducted experimental studies. Considerable variation exists between the observations of the investigators and there is some variation in the general recommendations by the respective investigators. One procedure outlined by Bird, Stewart and Lightfoot (1960) is based on the Ergun (1952) equation for estimating the friction factor in the Darcy equation:

$$F = \Delta p / \gamma = f(L/D_p)(V^2/2g) \quad (2.32)$$

where

F = friction loss due to resistance of bed of material per unit weight of fluid (J/N)

Δp = change in fluid pressure in passing through bed (Pa)

γ = specific weight of fluid (N/m³)

f = friction factor (dimensionless)

L = depth of bed of material (m)

D_p = diameter of particles in bed (m)

V = volume flow rate of fluid per unit cross-sectional area of bed,
sometimes called superficial velocity (m³/s m² or m/s)

g = acceleration due to gravity (m/s²)

When the particles are not spheres, the particle diameter used in equation 2.32 and in the Reynolds number is computed as:

$$D_p = 6v_p/s_p \quad (2.33)$$

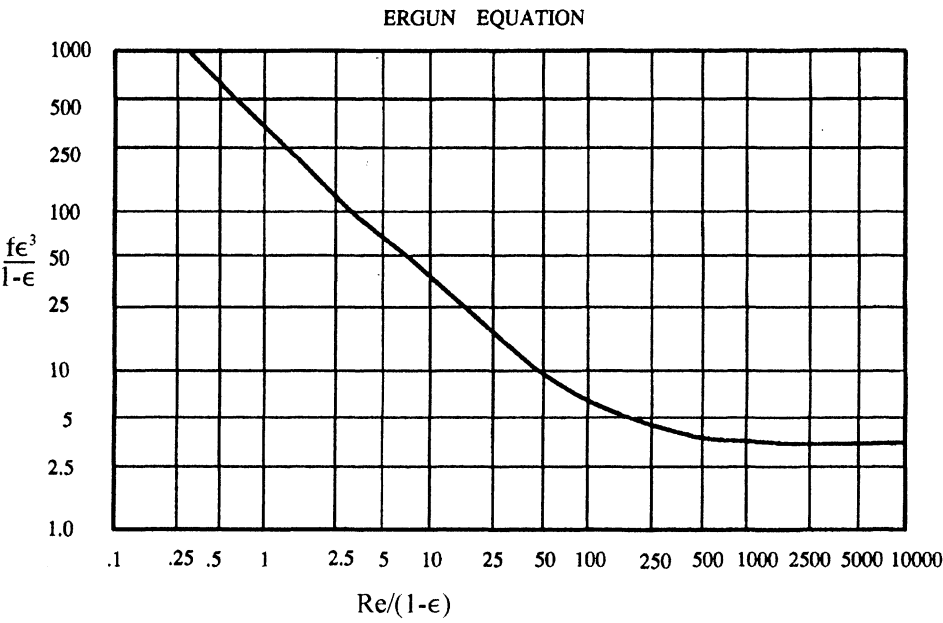


Figure 2.8. Plot of the Ergun equation for friction factor for fluid flowing through a bed of particles.

where

- v_p = particle volume (m^3)
- s_p = particle surface area (m^2)

The Ergun equation for estimating the friction factor f as a function of the Reynolds number and the voidage, ϵ_p , of the bed is as follows:

$$f = ((1 - \epsilon_p)/\epsilon_p^3)(300(1 - \epsilon_p)/Re + 3.5) \tag{2.34}$$

where ϵ_p = bed voidage (dimensionless).

A plot of $f\epsilon_p^3/(1 - \epsilon_p)$ versus $Re/(1 - \epsilon_p)$ results in the single curve shown in figure 2.8. Laminar flow is expected when the $Re/(1 - \epsilon_p)$ term is less than approximately 10.

Hukil and Ives (1955) suggested the following relationship for describing the pressure loss data for agricultural materials:

$$\Delta p/L = aV^2/\ln(1 + bV) \quad (2.35)$$

where

- Δp = pressure loss through bed (Pa)
- L = depth of bed of material (m)
- V = superficial velocity (m/s or m³/s m²)
- a = constant for material (Pa s²/m³)
- b = constant for material (m² s/m³)

Table 2-5 gives the values of a and b which best fit the data shown in figures 2.9 and 2.10. The experimental data on pressure drop through beds of material is for clean, loosely packed beds. Therefore, it is common practice to apply a correction factor to the estimates obtained from equation 2.35 in order to more nearly predict the pressure drop in a real situation where foreign material is in the bed and the material may be more densely packed due to handling operations. Though the correction factor may vary widely for given situations, a value of 1.5 is often used for engineering design. Then:

$$\Delta p = aC_fLV^2/\ln(1 + bV) \quad (2.36)$$

where C_f = correction factor (dimensionless).

Once the pressure drop is computed from equation 2.36, the friction term F of the Bernoulli equation may be determined as:

$$F = \Delta p/\gamma \quad (2.37)$$

While the empirical relationship represented by the Hukil and Ives (1955) equation is quite useful in predicting the pressure drop through agricultural products for the conditions under which experimental data have been obtained, it does not allow for predictions under varying voidage fractions, particle diameters, or fluids. Further work on development of an Ergun-type relationship to satisfactorily describe resistance in beds of agricultural products appears needed.

2.15 Energy Loss Due to Flow Through Floors

The perforated floor or wall which retains an agricultural product being dried or aerated offers resistance to air flow in addition to the resistance of the material. Henderson (1943) found the following experimental relationship for perforated floors:

$$F = (1.071 \text{ Pa s}^2/\text{m}^2)(V/O_f)^2/(\rho g) \quad (2.38)$$

where

- F = friction loss due to floor (J/N)
- V = superficial velocity (m/s)

O_f = percent floor opening expressed as a decimal
 ρ = density of fluid (kg/m^3)
 g = acceleration due to gravity (m/s^2)

When a bed of material is placed on a perforated floor, the effective amount of floor opening is decreased. Theoretically, we would expect a decrease in the effective area by a fraction equal to the voidage fraction of the material. Henderson (1943) confirmed this expectation for shelled corn having a voidage fraction of 40%. Assuming this condition to hold for all material, the expression for energy loss through a perforated floor supporting a bed of material becomes:

$$F = (1.071 \text{ Pa s}^2/\text{m}^2) (V/(O_f \varepsilon_p))^2 / (\rho g) \quad (2.39)$$

where ε_p = voidage fraction of the material.

Nomenclature

A	area (m^2)
C_f	correction factor for pressure drop through beds (dimensionless)
D	diameter (m)
D_h	hydraulic diameter (m)
D_p	diameter of particles (m)
e	efficiency of pump, fan, etc. (dimensionless decimal)
F	energy loss to friction per unit weight of fluid (J/N)
f	friction factor (dimensionless)
g	acceleration due to gravity (9.81 m/s^2)
h	elevation above a reference plane (m)
K	friction loss factor (dimensionless)

L	length (m)
\dot{m}	mass flow rate (kg/s)
O_f	percent opening in floor (decimal)
p	pressure (Pa)
p_{atm}	atmospheric pressure (Pa)
P	force (N)
P_r	power required by pump, fan, etc. (W)
P_s	power supplied to the fluid system (W)
Q	volumetric flow rate (m ³ /s)
r	radius (m)
R_h	hydraulic radius (m)
Re	Reynolds number (dimensionless)
s_p	particle surface area (m ²)
t	time (s)
V	velocity (m/s)
v_p	particle volume (m ³)
W	work done on fluid per unit weight of fluid (J/N)
y	separation distance (m)
ε	roughness factor (m)
ε_p	voidage fraction (dimensionless)
γ	specific weight of fluid (N/m ³)
μ	dynamic viscosity of fluid (Pa s)
ν	kinematic viscosity (m ² /s)
ϕ	angle of repose (rad)
ρ	fluid density (kg/m ³)

Displacement Pumps

Displacement pumps may be further subdivided into reciprocating or rotary-type pumps. Each of these types are often referred to as positive displacement pumps and are used in situations where constant and/or highly predictable flow rates are desired. They may also be capable of providing flow against relatively high heads.

4.2 Reciprocating or Piston Pumps

Reciprocating pumps (also called piston pumps) consist of a crank and connecting rod which are driven by a motor to move a piston or plunger as shown in figure 4.1. The piston or plunger may be used to directly move the fluid by providing appropriate

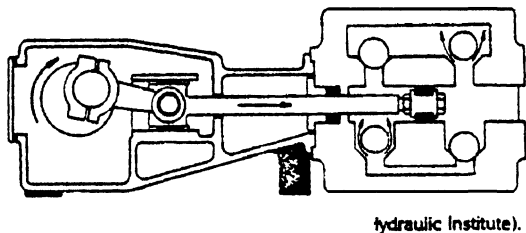


Figure 4.1. Piston-packed pump.

intake and exhaust valves or it may be connected to a diaphragm with a known volumetric displacement for each stroke of the piston. Volumetric efficiencies above 97% and overall mechanical efficiencies around 90% are common for these pumps.

Piston pumps manufactured for reliable, heavy-duty service at pressures of 35 MPa or more are quite common. However, when the pressures of operation exceed 105 MPa, the piston velocities have to be considerably reduced to get satisfactory packing life and to decrease the number of pressure reversals and cyclic stresses that are involved in fatigue failures. Consequently, as a result of this piston velocity reduction the piston sizes that will be required and the subsequent loads on them will make the more common crank-driven piston pumps impractical.

In an operation which involves continuous full flow operation at pressures of 10 MPa or more, the energy savings that can result from the relatively high overall efficiency of a piston pump can be considerable. The dairy industry takes advantage of the high efficiency of piston pumps when using them with two-stage valves attached as homogenizers in the pasteurization of milk. Also, piston pumps do not need priming, and because of the relatively slow speed of operation the parts that are subjected to the most wear, like gaskets, valves, pistons, etc., are simple and easy to maintain.

The two strokes that comprise a complete cycle of the piston pump are demonstrated in figure 4.2. Each cylinder of this type pump requires the operation of two valves; one for the suction stroke and one for the discharge stroke with only the discharge stroke being productive as far as product flow through the system is concerned. Due to this lack of productive flow during the suction stroke of an individual cylinder, multiplexing or coupling of two or more liquid pumping cylinders is the general rule in piston pump design. The most inexpensive way to accomplish this coupling is to use a double acting piston. This results in a pump which discharges for the full 360 degree revolution of

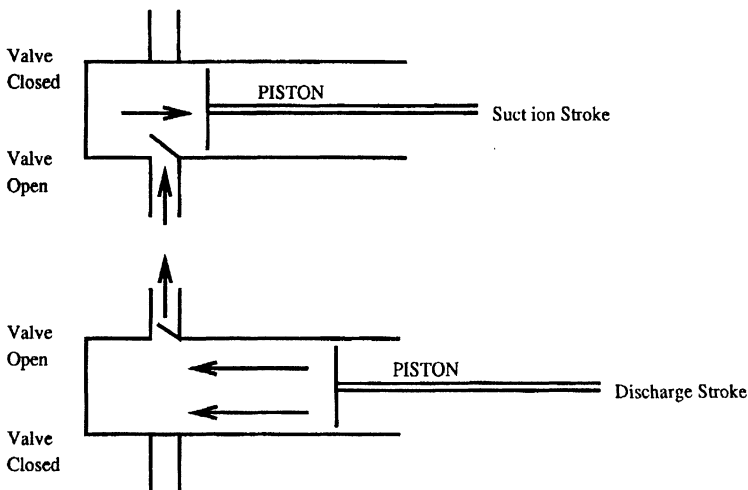


Figure 4.2. Plunger pump. The valve and piston action of a piston-type pump or both the suction and discharge strokes.

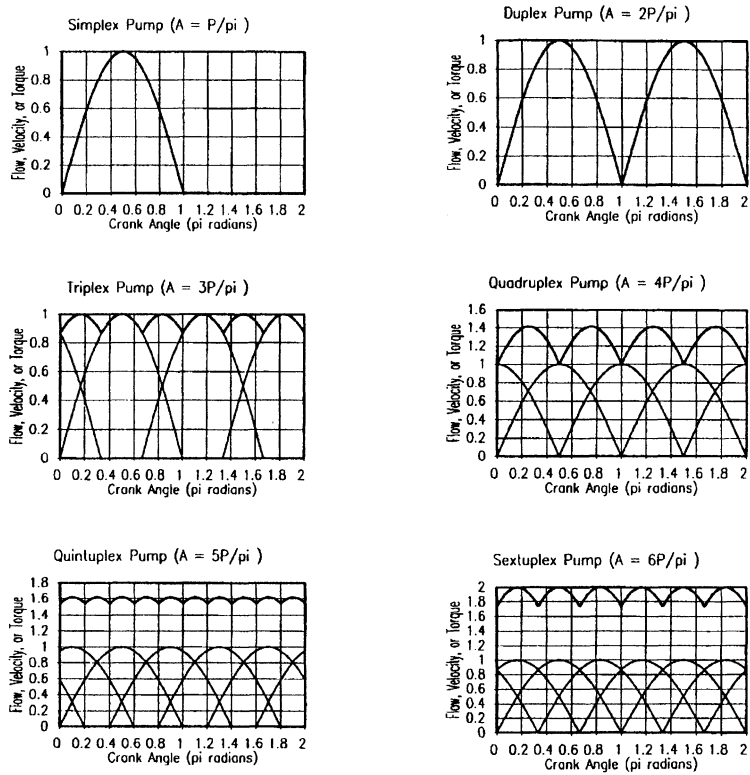


Figure 4.3. Flow-pressure-torque curves on multiplex pumps (A = average for multiplex pump, P = peak for single cylinder).

the crankshaft. The major limitation of this configuration is that duplexing is all that can be accomplished with it; therefore, multiplexing of single acting pistons is the most common configuration for piston pumps. Duplexing individual cylinders in this manner, so that one piston is 180 degrees out of phase with the other, allows the doubling of efficiency and flow rate without requiring an increase in torque from the crankshaft, since the available power from the crankshaft is used for productive forward flow in the system 100% of the time rather than 50% of the time. With duplexing there is still considerable fluctuation in flow rate and pressure as indicated in figure 4.3. Thus, the addition of more individual cylinders working in concert is beneficial to the stability of flow obtained from a piston pump. It is important to note that if stability of flow is of prime importance, the total number of individual cylinders in the pump should be an odd number. However, the addition of an odd cylinder to a piston pump entails an increase in the torque required from the crankshaft, as opposed to the lack of an increase in crankshaft torque for the duplex arrangement. So, the odd cylinder stabilizes flow but requires additional input energy. This trade off situation requires consideration when one is deciding which piston pump is appropriate for a particular application.

4.3 Rotary Pumps

Rotary pumps are inexpensive and simple to construct. If constructed and maintained with very close tolerances, the volumetric efficiency is high (e.g., 95%) and high pressures can be produced. Mechanical efficiencies may be 90% or more under the best conditions.

There are several types of rotary pumps. Each one is used for a different need or purpose. These types include gear, lobe, screw, and vane.

The gear pump is probably the most common of the rotary pumps. It works with either external or internal gears as shown in figures 4.4 and 4.5. One gear is driven while

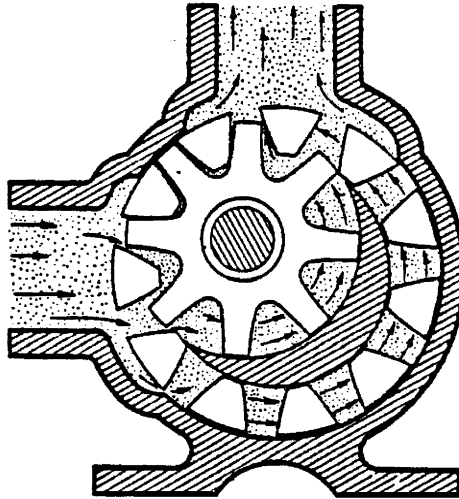


Figure 4.4. Internal-gear rotary pump.

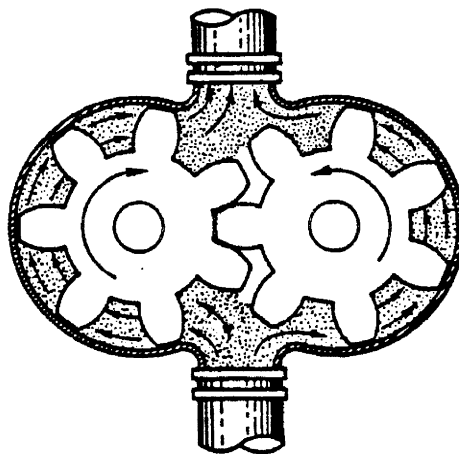


Figure 4.5. External-gear rotary pump.

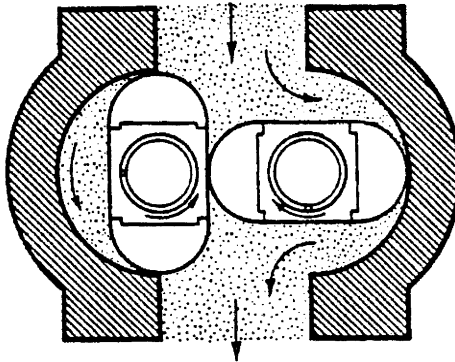


Figure 4.6. Lobe pump, used for both gases and liquids.

the other rotates freely. A partial vacuum is created by the unmeshing of the gears near the inflow of the fluid. This vacuum draws the fluid into the pump. As the gears mesh together, the fluid is pushed out the other side. The gears need constant lubrication, therefore viscous substances are better for gear pumps. They are especially suited for viscous substances such as ice-cream mix, molasses, and oils. Only liquid or liquid gas mixtures can be used since the gear pump is subject to clogging. Though classified as displacement pumps which implies a pulsating flow, gear pumps provide relatively constant outputs.

Figure 4.6 shows a lobe pump. This pump type uses two, three, or four lobes in place of gears. Otherwise, the principles behind the operation of the lobe pump are basically

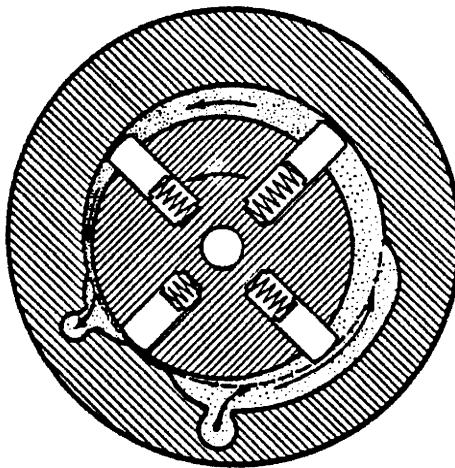


Figure 4.7. Vane pump, used for both gases and liquids

the same as for the gear pump. The two rotors on the lobe pump are driven—none of the lobes rotate freely. The lobe pump produces more of a pulsating output than the gear pump. It can pump both liquids and gases.

The screw pump consists of a helical screw which rotates within a fixed casing and transfers fluid from the inlet to the outlet. Since the screw pump does not operate with any kind of rotating gears, it can pump liquids with solid particles intermixed. The screw pump also produces a very steady output.

Figure 4.7 shows a vane pump. This pump operates with sliding vanes and hinged parts to maintain the seal that is needed. The vane is used in evacuating processes where a vacuuming effect is needed, e.g., a milking machine. The vane pump moves vapor and liquids only—no solid mixtures. It also provides a relatively steady output.

4.5 Jet Pumps

The jet pump is a special effect pump shown elementarily in figure 4.12 which operates on the velocity energy of a jet of fluid. Water (or other fluid, either compressible or

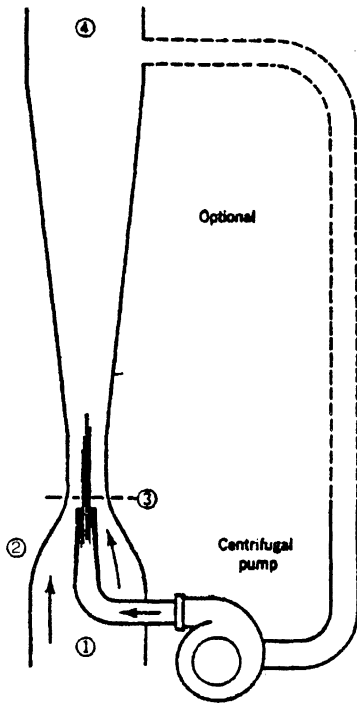


Figure 4.12. Schematic elements of a jet pump.

incompressible) is forced through a jet or nozzle of such dimensions that all or nearly all the energy involved is converted into velocity energy. This energy, which is directional, is applied to the fluid to be moved. The jet is produced by recirculating a portion of the liquid or gas in those cases where the material will not damage the pump. Jet pumps are frequently used for pumping sumps or processing residues that contain solid matter or chemically active materials that would not pass through a mechanical pump satisfactorily. For example, if the material is a gas, water or air is provided from an external source to supply the jet energy. The diluted mixture is discarded.

Jet pump theory (Stepanoff, 1957) is rational and straightforward but too involved to include in this text. However, the basic consideration is the conservation of energy, which may be expressed as:

$$H_1 V_1 A_1 \gamma_1 + H_2 V_2 A_2 \gamma_2 = H_4 V_4 A_4 \gamma_4 + P_f \quad (4.27)$$

where

H = total fluid hydraulic head (m)

V = fluid velocity (m/s)

A = cross-sectional area (m²)

γ = specific weight of fluid (N/m³)

P_f = power loss due to friction and turbulence in the mixing process (W)

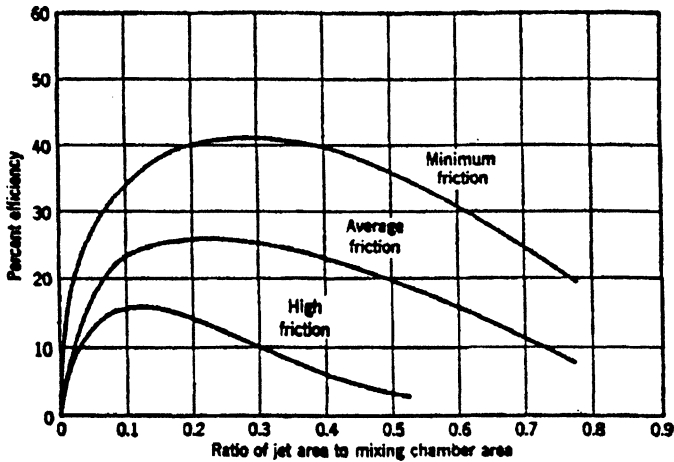


Figure 4.13. Jet-pump efficiency related to the relative jet area and friction in the jet unit.

The total hydraulic head of the fluid streams can be made up of any combination of elevation, pressure, and velocity heads. However, H_2 , the head of the recirculated or external fluid, must be predominately velocity head in order to transfer the energy to the fluid entering at 1 in figure 4.12. The efficiency may be expressed as:

$$e_j = (H_4 V_4 A_4 \gamma_4 - H_1 V_1 A_1 \gamma_1) / (H_2 V_2 A_2 \gamma_2) \quad (4.28)$$

where e_j = efficiency of the jet portion of pump (decimal).

The efficiency is closely related to the ratio of the nozzle area to the mixing-cylinder area and the friction in the system. The relationship of these factors is shown in figure 4.13. These are theoretical curves that have been substantiated by observation.

In spite of its low efficiency, the simplicity of the jet pump, its freedom from moving parts, its ability to pump materials of sludge consistency, and its low initial cost fit it for use in situations where other pumping devices would be impractical.

4.6 Air Lift Pumps

The air lift pump is a special effect pump having the arrangement illustrated in figure 4.14. Compressed air is released through a "foot-piece", or air diffuser, at the bottom of the vertical pipe called the eductor pipe. A mixture of air bubbles and liquid to be lifted is created which has a density less than that of the liquid alone. This lower density mixture rises in the pipe and is discharged at a point above the level of the liquid. The weight of the column of air and liquid, $h_f + h_s$, is equal to the weight of the column of liquid, h_s . For maximum efficiency of the airlift pump the air bubbles should be small. The small bubbles exert no appreciable lifting force on the liquid.

Air lifts are the most simple and foolproof type of pump and, in operation, give the least trouble because there are no remote or submerged moving parts. They can be

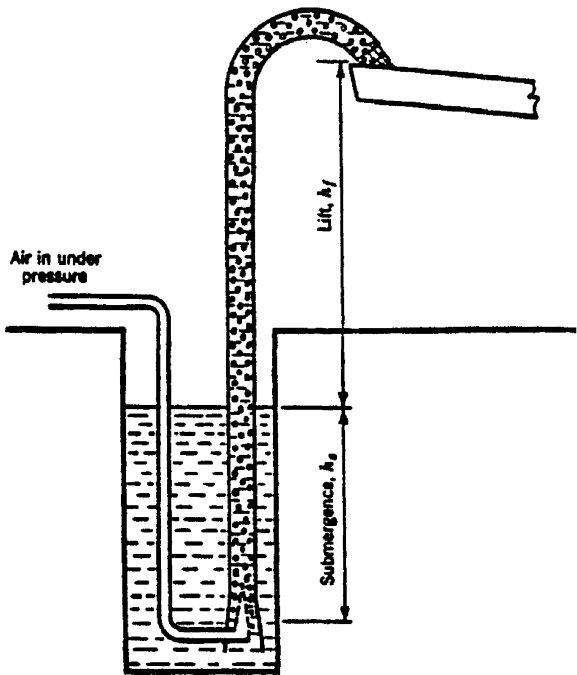


Figure 4.14. The air lift.

operated successfully in holes of any practicable size, and they can be used in crooked holes not suited to any other type of pump.

The principal disadvantages of air lifts are the necessity for making a deeper well than is required for other pump types, the intermittent nature of the flow, and the relatively low efficiencies obtained. Babbitt et al. (1967) give approximate efficiencies to be expected for air lift pumps while pumping water. Table 4-1 summarizes the expected efficiencies

Table 4-1. Effect of submergence on efficiencies of air lift at Hattiesburg, Miss.

Ratio, h_s/h_f	Submergence Ratio, $h_s/(h_s + h_f)$	Percent Efficiency
8.70	0.896	26.5
5.46	0.845	31.0
3.86	0.795	35.0
2.91	0.745	36.6
2.25	0.693	37.7
1.86	0.650	36.8
1.45	0.592	34.5
1.19	0.544	31.0
0.96	0.490	26.5

Table 4-2. Some recommended submergence percentages for air lifts

Lift, m	Submergence Percentage
Up to 15	70–66
15–30	66–55
30–60	55–50
60–90	50–43
90–120	43–40
120–150	40–33

as a function of the ratio, h_s/h_f , and the submergence ratio, $h_s/(h_s + h_f)$. Table 4-2 gives submergence percentages recommended for air lifts.

The volume flow rate of air which is required to give the desired liquid flow rate may be estimated from the following empirical equation:

$$Q_a = Q_w(h_f + h_v)/(22.9E \log((h_s + 10.4)/10.4)) \quad (4.29)$$

where

- Q_a = volumetric flow rate of air (m^3/s)
- Q_w = volumetric flow rate of liquid (m^3/s)
- h_f = lift required (m)
- h_v = velocity head of liquid at discharge (m)
- E = pump efficiency estimated from table 4-1 (decimal)
- h_s = submergence (m)

The air pressure which must be provided at the submerged end of the air pipe or foot piece is equal to the pressure exerted by a column of the fluid equal to h_s .

Some excess air capacity should be provided, because if the free-liquid surface in the well should fall more than anticipated after prolonged pumping, more air will be required to maintain the discharge.

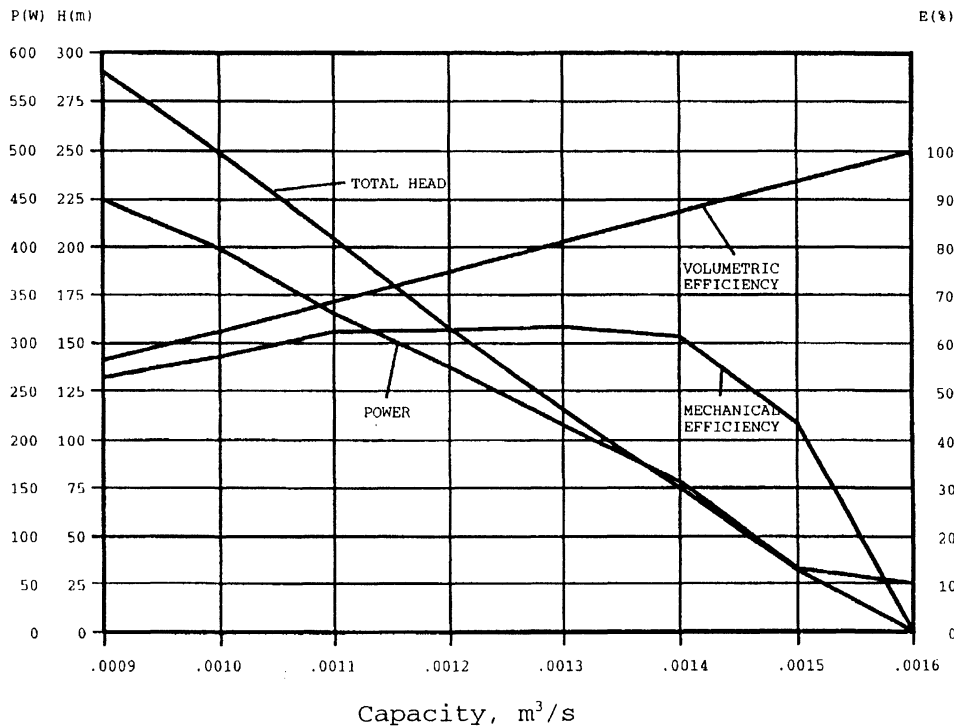


Figure 4.17. Performance of a gear pump. Fluid density is 928 kg/m^3 .

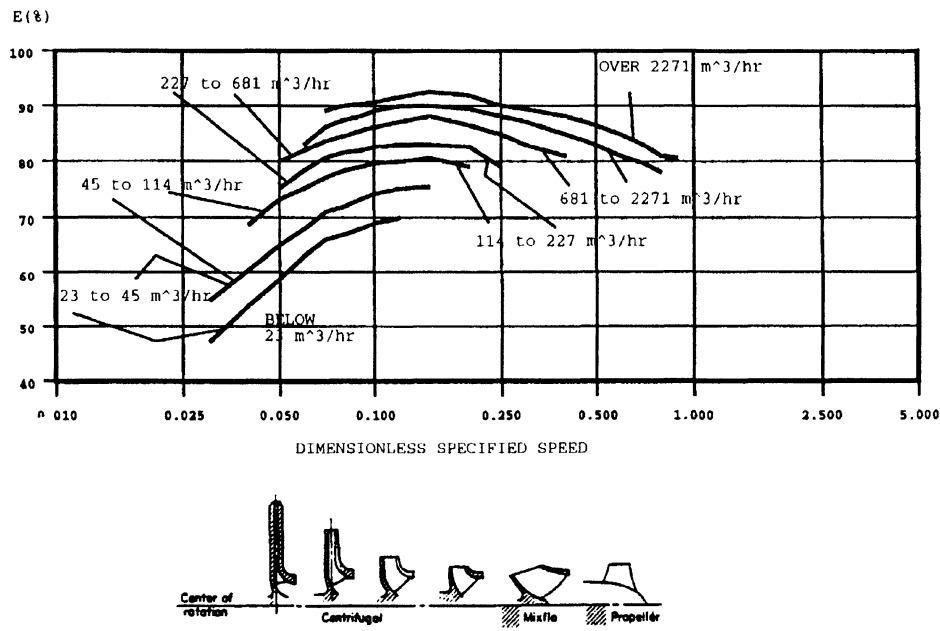


Figure 4.18. Approximate relative impeller shapes and efficiency variations with specific speed.

Chapter 5, Fans

A set of fan performance curves consists of plots of total pressure, static pressure, power, total efficiency, and static efficiency versus the volumetric flow rate of the fan. If either the static pressure or total pressure curve is given or experimentally determined for a fan of known outlet area, then the other can be determined from the following

relationship:

$$p_t = p_s + (v^2 \rho)/2 \quad (5.1)$$

where

p_t = total pressure (Pa)

p_s = static pressure (Pa)

v = air velocity at fan outlet (m/s)

ρ = air density (kg/m³)

The velocity of air at the fan outlet is determined from the volumetric flow rate and outlet area by:

$$v = Q/A \quad (5.2)$$

where

Q = volumetric flow rate (m³/s)

A = outlet area of fan (m²)

If any one of the power, static efficiency, or total efficiency curves is given or determined, the others can then be calculated from the following relationships:

$$e_t = p_t Q/P \quad (5.3)$$

and

$$e_s = p_s Q/P \quad (5.4)$$

where

e_t = total efficiency (decimal)

p_t = total pressure (Pa)

Q = volumetric flow rate (m³/s)

e_s = static efficiency (decimal)

p_s = static pressure (Pa)

P = power input to fan (W)

Thus, a minimum of two curves may be used to calculate a set of five performance curves.

5.11 Sound Power Level Ratings

The sound emitted by a fan is an inevitable byproduct of the energy-transfer process. However, careful selection of fans so that they operate at quieter points on their sound power level curves, will minimize noise.

Sound power level is expressed in decibels based on the following relationship:

$$L = 10 \log(W / W_o) \tag{5.5}$$

where

- L = sound power level (dB)
- W = power (W)
- W_o = reference power = 1 pW = 1×10^{-12} W

Table 5-1 gives typical sound powers and sound power levels for some familiar sources. Table 5-2 lists permissible noise exposures as specified by the Occupational Safety and Health Act (OSHA). Note that the permissible sound levels are expressed in terms of equivalent A-weighted sound levels (dBA). The procedure for determining the equivalent A-weighted level is discussed in Chapter 4 of the reference edited by Jorgensen (1983). It is in effect a procedure which adjusts the sound level at a frequency other than 1000 Hz to an equivalent level at 1000 Hz. This is done because humans react less in terms of loudness to low frequency and high frequency sounds than they do to sounds of medium frequency content, when all are at the same level.

Table 5-1. Typical sound power and sound power levels

Power (watts)	Power Level (dB re 1 pW)	Source (long time avg.)
100 000	170	Jet airplane
10 000	160	
1 000	150	
100	140	
10	130	Large orchestra
1	120	
0.1	110	Blaring radio
0.01	100	
0.001	90	Shouting
0.000 1	80	
0.000 01	70	Conversational Speech
0.000 001	60	
0.000 000 1	50	Small electric clock
0.000 000 01	40	
0.000 000 001	30	Soft whisper

Table 5-2. Permissible noise exposures

Duration per day, hours	Sound Level, dBA
8	90
6	92
4	95
3	97
2	100
1.5	102
1	105
0.5	110
0.25 or less	115

If the sound power of two or more sources is combined, the overall sound power level is given by:

$$L_c = 10 \log(10^{(L_1/10)} + 10^{(L_2/10)} + \dots + 10^{(L_n/10)}) \tag{5.6}$$

where

- L_c = overall sound power level (dB)
- L_1, L_2, \dots, L_n = sound power levels of individual sources (dB)

5.12 Specific Speed

The same non-dimensional specific speed used for pumps is useful in characterizing fans. If the total head in equation 4.33 is replaced by its equivalent, (p_t/γ) , then the specific speed equation becomes:

$$N_s = N Q^{1/2} \rho^{3/4} / p_t^{3/4} \tag{5.7}$$

where

- N_s = specific speed (dimensionless)
- N = fan speed (rev/s)

Q = volumetric flow rate (m^3/s)

ρ = air density (kg/m^3)

p_t = total pressure (Pa)

5.17 Fan Laws

As was the case for centrifugal pumps in Chapter 4, the performance of geometrically similar fans operating at speeds other than those for which tests were conducted may be estimated by use of the affinity laws. Equations 4.30, 4.31, and 4.32 may be stated for fans as follows:

$$Q_1/Q_2 = (N_1/N_2)(D_1/D_2)^3(K_2/K_1) \quad (5.8)$$

$$p_{t1}/p_{t2} = (N_1/N_2)^2(D_1/D_2)^2(\rho_1/\rho_2)(K_2/K_1) \quad (5.9)$$

$$P_1/P_2 = (N_1/N_2)^3(D_1/D_2)^5(\rho_1/\rho_2)(K_2/K_1) \quad (5.10)$$

where

Q = volumetric flow rate (m^3/s)

D = fan diameter (m)

N = fan speed (rev/s)

K = compressibility factor (dimensionless)

ρ = air density (kg/m^3)

p_t = total pressure (Pa)

P = fan input power (W)

A fourth relationship may be given to express the change in sound power level:

$$L_1 = L_2 + 70 \log(D_1/D_2) + 50 \log(N_1/N_2) + 20 \log(\rho_1/\rho_2) \quad (5.11)$$

where L = sound power level (dB).

5.18 Compressibility

The effects of compressibility are accounted for in the fan laws by the inclusion of a compressibility factor K . This coefficient may be approximated by the following relationship:

$$K = c((p_{t2}/p_{t1})^{1/c} - 1)/(p_{t2}/p_{t1} - 1) \quad (5.12)$$

where

K = compressibility factor (dimensionless)

c = $ne_t/(n - 1)$ (dimensionless)

n = isentropic exponent = 1.4

e_t = total efficiency of fan (decimal)

p_{t1} = absolute total pressure at fan entrance (Pa)

p_{t2} = absolute total pressure at fan exit (Pa)

The system characteristic curves are based on the relationship:

$$p_t = C Q^2 \quad (5.13)$$

where

p_t = total pressure (Pa)

Q = volumetric flow rate (m^3/s)

C = proportionality constant = 15, 10, and 5 $\text{Pa s}^2/\text{m}^6$ for systems A , B , and C , respectively

Nomenclature

A	outlet area of fan (m^2)
D	diameter (m)
e_s	static efficiency (decimal)
e_t	total efficiency (decimal)
L	sound power level (dB)
n	isentropic exponent = 1.4
N	fan speed (rev/s)
N_s	specific speed (dimensionless)
P	power input to fan (W)
p_s	static pressure (Pa)
p_t	total pressure (Pa)
Q	volumetric flow rate (m^3/s)
v	air velocity (m/s)
W	sound power (W)
W_o	reference power level = $1 \text{ pW} = 1 \times 10^{-12} \text{ W}$
γ	specific weight (N/m^3)
ρ	density of air (kg/m^3)

Table 6-1. Coefficients of thermal conductivity of various materials

Material	Apparent Density (kg/m ³)	Temperature (C)	k (W/mK)
Air		0	0.0242
Asbestos, cement boards	1920	20	0.74
Asbestos sheets	889	51	0.166
Asbestos	580	0	0.156
	580	100	0.192
Aluminum		0	202
Aluminum foil, 7 air spaces per 6.4 cm	3.2	38	0.043
Brick, building		20	0.692
Cardboard, corrugated			0.064
Concrete 1:4 dry			0.76
Concrete, stone			0.93
Copper, pure		18	388
		100	377
Cotton wool	80	30	0.042
Cork, board	160	30	0.043
Cork, ground	150	30	0.043
Diatomaceous earth	444	204	0.114
	444	871	0.159
Fiber insulating board	237	21	0.048
Glass, boro-silicate	2230	30–75	1.09
Glass, soda			0.5–0.76
Glass, window			0.5–1.06
Ice	921	0	2.2
Iron, wrought		18	60.4
		100	59.9
Iron, cast		54	47.8
		102	46.4
Mill shavings			0.057–0.09
Mineral wool	151	30	0.0389
	316	30	0.042
Sawdust	190	21	0.05
Snow	556	0	0.47
Steel, mild		18	45.3
		100	44.8
Steel, stainless (18-8)		500	21.5
Water		0	0.571
Wood shavings	140	30	0.059
Wood, across grain, balsa	110–130	30	0.043–0.5
oak	825	15	0.21
white pine	540	15	0.151
Wool, animal	110	30	0.036

Table 6-2. Typical resistance values for building materials

	Resistivity (mK/W)	Resistance (m ² K/W)
Air Spaces		
Bounded by ordinary materials		
1.9 cm or more in width		
Vertical		0.16
Horizontal, heat flow up		0.13
Horizontal, heat flow down		0.19
Bounded by aluminum foil		
Vertical		0.38
Horizontal, heat flow up		0.15
Horizontal, heat flow down		0.40
Space 3.81 cm or more divided by material reflective on both sides		
Vertical		0.76
Horizontal, heat flow up		0.65
Horizontal, heat flow down		1.96
30° slope, heat flow up		0.70
30° slope, heat flow down		1.35
Exterior finishes, frame walls		
Brick veneer, 10.2 cm thick (nominal)		0.08
Stucco	0.55	
Wooden shingles		0.14
White pine and redwood	8.93	
Southern yellow pine lap siding	8.70	
1.6 cm thick	8.70	0.14
Wood sheathing, paper and siding		
Fir sheathing—2 cm yellow pine lap siding		0.35
Asbestos shingles		0.03
Plywood siding, three-ply, 1 cm		0.07
Stone veneer	0.58	
Insulating materials		
Fill and blankets, mineral or vegetable fiber or animal hair	25.64	
Corkboard	23.26	
Insulating board	20.83	
Sawdust and shavings	16.95	
Macerated paper	25.64	
Shredded wood and cement, slab	15.15	
Shredded redwood bark (80 kg/m ³)	27.03	
Paper and asbestos fiber	25.00	
Mineral wool made from rock, slag or glass	25.64	
Vermiculite, expanded mica	21.74	
Vermiculite, expanded	14.49	
Cotton, batt or blanket	28.57	
Balsa wood, 144 kg/m ³	18.18	
Kapok, 16 kg/m ³	28.57	

Table 6-2. (Continued)

	Resistivity (mK/W)	Resistance (m ² K/W)
Interior finishes		
Composition wallboard	13.89	
Gypsum plaster, 1.3 cm thick	2.10	0.03
Gypsum wallboard, 1 cm thick		0.05
Gypsum lath and plaster, 1.3 cm plaster		0.07
Insulating fiberboard (1.3 cm)		0.27
Insulating board lath (1.3 cm) and plaster (1.3 cm)		0.29
Insulating board lath (2.5 cm) and plaster (1.3 cm)		0.57
Metal lath and plaster (1.9 cm)		0.04
Plywood, three-ply (0.6 cm)	6.94	0.04
Wood lath and plaster		0.07
Cement plaster	0.87	
Masonry materials		
Brick, common (10.2 cm)	1.39	0.14
Brick, face (10.2 cm)	0.76	0.08
Cement mortar	0.58	
10.2 cm hollow clay tile		0.18
15.2 cm hollow clay tile		0.28
20.3 cm hollow clay tile		0.29
25.4 cm hollow clay tile		0.30
30.5 cm hollow clay tile		0.44
Concrete, lightweight aggregate	2.77	
Concrete, cinder aggregate	1.39	
Concrete, ordinary	0.87	
Concrete, sand and gravel aggregate	0.58	
10.2 cm concrete blocks, hollow cinder aggregate		0.18
20.3 cm concrete blocks, hollow lightweight aggregate		0.35
cinder aggregate		0.29
sand and gravel aggregate		0.18
20.3 cm painted concrete blocks		0.34
20.3 cm insulated concrete blocks cores filled with rockwool		0.65
30.5 cm concrete blocks, hollow lightweight aggregate		0.37
cinder aggregate		0.33
sand and gravel aggregate		0.22
cinder aggregate		0.33
sand and gravel aggregate		0.22
7.6 cm gypsum tile, hollow		0.29
10.2 cm gypsum tile, hollow		0.38
Tile and terrazzo	0.58	
Stone	0.58	
2 coats p.c. paint		0.03
Window glass	1.26	

Table 6-2. (Continued)

	Resistivity (mK/W)	Resistance (m ² K/W)
Roofing materials		
Asbestos shingles		0.03
Asphalt shingles		0.03
Built-up roofing (1 cm)		0.05
Heavy roll roofing		0.03
Slate (1.3 cm)	0.69	0.01
Wood shingles		0.14
Sheathing		
Asbestos-cement board	2.57	
Gypsum, 1.3 cm		0.06
Insulating fiberboard (2 cm)		0.42
Plywood, 3-ply, 0.8 cm thick		0.05
Wood (2 cm)	8.70	0.17
Wood (2.5 cm), soft	10.64	
hard	5.68	
Wood (2.5 cm) plus building paper		0.20
Surfaces		
Inside surfaces, Ordinary materials		
still air, vertical		0.11
horizontal, heat flow up		0.09
horizontal, heat flow down		0.15
Outside surfaces, Ordinary materials		
24.1 km/h wind		0.03
48.3 km/h wind		0.02
Reflective materials		
24.1 km/h wind		0.03

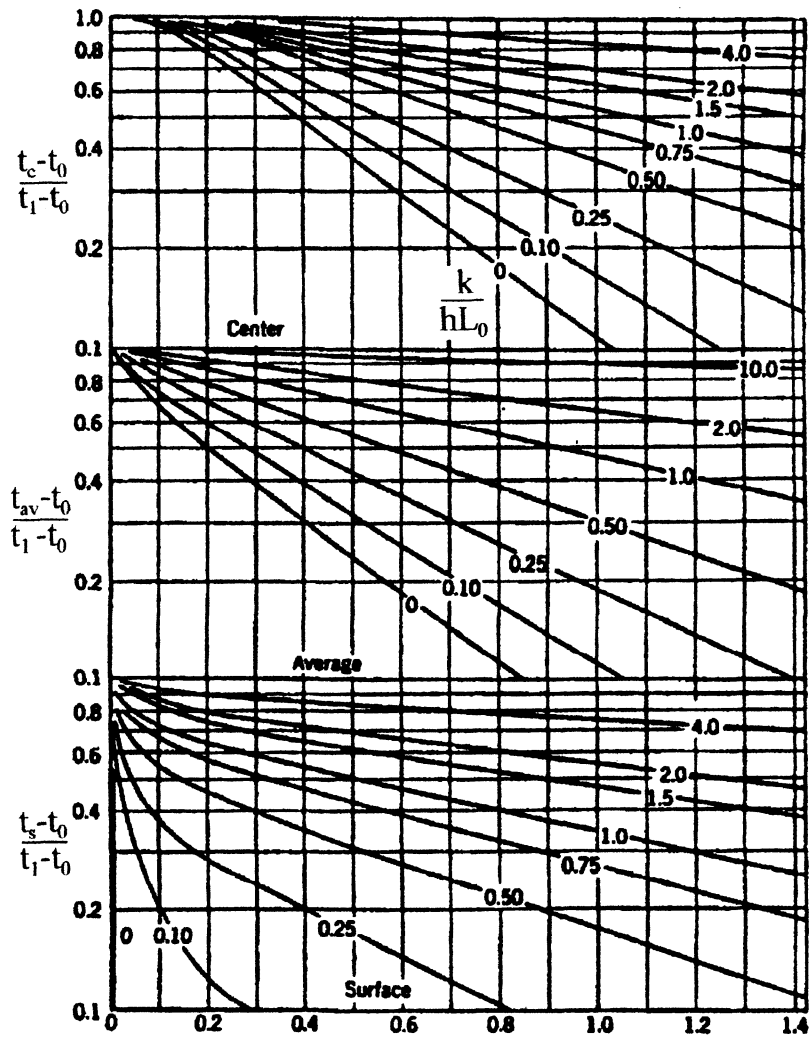


Figure 6.11. Transient temperatures in a slab.

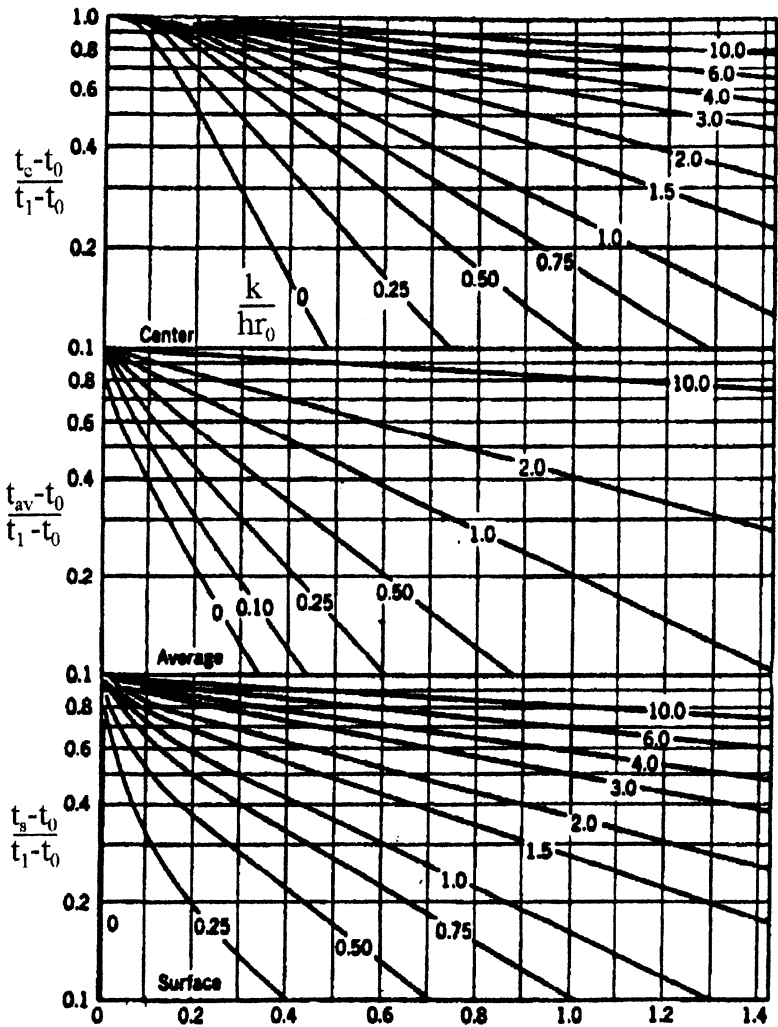


Figure 6.12. Transient temperatures in a cylinder.

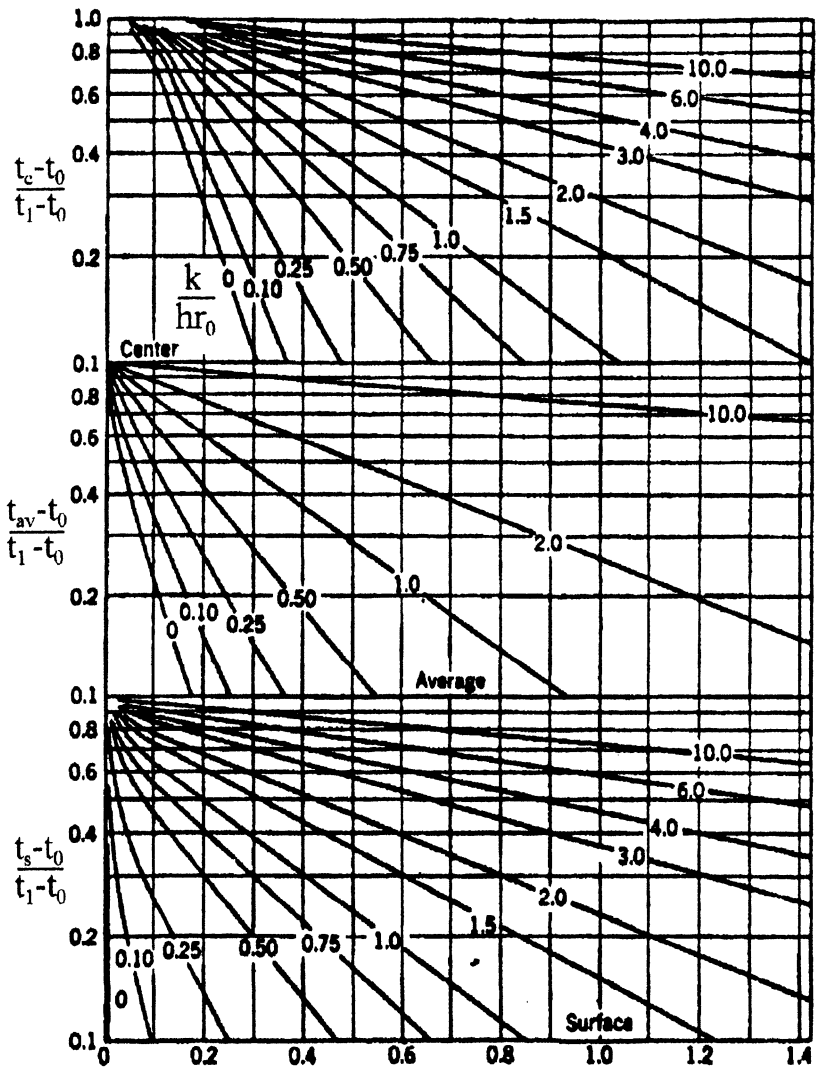


Figure 6.13. Transient temperatures in a sphere.

Chapter 7, Heat Transfer

For shapes other than cylinders, similar equations have been developed. For plane surfaces in air at 20 C:

1. Horizontal, heated facing upward, or cooled facing downward:

$$h = 1.32(\Delta t/L)^{0.25} \quad \text{Laminar range (small plates)} \quad (7.44)$$

$$h = 1.52(\Delta t)^{1/3} \quad \text{Turbulent range (large plates)} \quad (7.45)$$

2. Horizontal, heated facing downward, or cooled facing upward, square:

$$h = 0.59(\Delta t/L)^{0.25} \quad \text{Laminar range (small plates)} \quad (7.46)$$

3. Vertical surfaces:

$$h = 1.42(\Delta t/L)^{0.25} \quad \text{Laminar range (small plates)} \quad (7.47)$$

$$h = 1.31(\Delta t)^{1/3} \quad \text{Turbulent range (large plates)} \quad (7.48)$$

In each of these relationships the laminar range is considered to be the range where the product of the Grashof and Prandtl numbers is between 10^4 and 10^8 while the turbulent range has ($GrPr$) between 10^8 and 10^{12} .

Table 7-2. Properties of dry air at atmospheric pressure (101.35 kPa)*

Temperature <i>t</i> (C)	Density ρ (kg/m ³)	Dynamic Viscosity μ (Pa * s)	Kinematic Viscosity ν (m ² /s)	Specific Heat c_p (kJ/kg K)	Thermal Conductivity k (w/mK)	Thermal Diffusivity α (m ² /s)	Prandtl Number <i>Pr</i>
-70	1.7390	0.00001357	0.000007812	1.003	0.01878	0.00001054	0.7245
-60	1.6573	0.00001409	0.000008501	1.003	0.01956	0.00001115	0.7224
-50	1.5331	0.00001461	0.000009244	1.003	0.02034	0.00001282	0.7206
-40	1.5153	0.00001513	0.000009987	1.003	0.02111	0.00001388	0.7188
-30	1.4534	0.00001565	0.000010777	1.004	0.02189	0.00001502	0.7174
-20	1.3965	0.00001617	0.000011587	1.004	0.02267	0.00001618	0.7158
-10	1.3436	0.00001669	0.000012436	1.004	0.02345	0.00001740	0.7145
0	1.2946	0.00001719	0.000013295	1.004	0.02413	0.00001857	0.7155
10	1.2486	0.00001767	0.000014168	1.005	0.02404	0.00001981	0.7148
20	1.2059	0.00001816	0.000015061	1.005	0.02561	0.00002114	0.7125
30	1.1663	0.00001864	0.000015990	1.006	0.02634	0.00002246	0.7117
40	1.1293	0.00001912	0.000016965	1.006	0.02700	0.00002378	0.7110
50	1.0936	0.00001960	0.000017941	1.007	0.02778	0.00002523	0.7106
60	1.0604	0.00002005	0.000018916	1.007	0.02856	0.00002674	0.7072
70	1.0345	0.00002050	0.000019892	1.008	0.02918	0.00002811	0.7081
80	1.0021	0.00002091	0.000020888	1.009	0.02980	0.00002952	0.7080
90	0.9734	0.00002135	0.000021941	1.010	0.03055	0.00003108	0.7058
100	0.9470	0.00002179	0.000023024	1.011	0.03122	0.00003262	0.7057
110	0.9227	0.00002222	0.000024090	1.012	0.03185	0.00003412	0.7062
120	0.8996	0.00002263	0.000025166	1.013	0.03254	0.00003572	0.7047
130	0.8770	0.00002304	0.000026281	1.014	0.03326	0.00003741	0.7025
140	0.8557	0.00002345	0.000027401	1.016	0.03389	0.00003901	0.7027
150	0.8355	0.00002386	0.000028552	1.017	0.03451	0.00004063	0.7032
160	0.8153	0.00002431	0.000029806	1.018	0.03513	0.00004232	0.7044
170	0.7980	0.00002472	0.000030968	1.020	0.03576	0.00004393	0.7049
180	0.7807	0.00002513	0.000032191	1.022	0.03638	0.00004561	0.7055
190	0.7646	0.00002551	0.000033368	1.023	0.03700	0.00004731	0.7055
200	0.7487	0.00002590	0.000034606	1.025	0.03763	0.00004905	0.7057
210	0.7328	0.00002629	0.000035884	1.027	0.03825	0.00005084	0.7058
220	0.7176	0.00002666	0.000037161	1.029	0.03887	0.00005266	0.7056
230	0.7032	0.00002703	0.000038454	1.031	0.03950	0.00005451	0.7055

* Adapted from data given by M. Jakob and G. A. Hawkins. 1957. *Elements of Heat Transfer*, 3rd Ed. New York: John Wiley & Sons, Inc.

7.7 Experimental Equations for Forced Convection

Following are a few of many equations which have been developed experimentally relating the dimensionless ratios of equation 7.39 for forced convection in different geometries. For the average heat transfer coefficient in long pipes to a fluid flowing within the pipe the equation is:

$$Nu = 0.023(Re)^{0.8}(Pr)^{0.4} \quad (7.49)$$

for Reynolds numbers above 2300 to insure turbulent flow.

For forced convection to gases flowing normal to single cylinders in the range of Re from 0.1 to 1 000:

$$Nu(1/Pr)^{0.3} = 0.35 + 0.47(Re)^{0.52} \quad (7.50)$$

while from $Re = 1\,000$ to $Re = 50\,000$:

$$Nu = 0.26(Re)^{0.6}(Pr)^{0.3} \quad (7.51)$$

For liquids flowing normal to single cylinders with Re between 0.1 and 200:

$$Nu = 0.86(Re)^{0.43}(Pr)^{0.3} \quad (7.52)$$

while for Re above 200, equation 7.50 is recommended.

For spheres, in the range of $Re = 20$ to $Re = 150\,000$ the equation is:

$$Nu = 0.36(Re)^{0.6}(Pr)^{0.3} \quad (7.53)$$

In each of these equations, the fluid properties are to be evaluated at the film temperature, the arithmetic average of surface and bulk mean fluid temperatures. Additional experimental equations are given by ASHRAE (1997) and various other heat transfer references.

Nomenclature

A	area (m^2)
B	buoyancy (N/m^3)
Bo	Biot Number (dimensionless)
c_p	specific heat (J/kg K)
E	heat exchanger effectiveness ratio (dimensionless)
g	acceleration due to gravity (m/s^2)
Gr	Grashof Number (dimensionless)
h	convective heat transfer coefficient ($\text{W/m}^2\text{K}$)
k	thermal conductivity (W/mK)
l	length (m)
Nu	Nusselt Number (dimensionless)
NTU	number of transfer units (dimensionless)
Pr	Prandtl Number (dimensionless)
q	heat flow (W)
q'	heat transfer per unit length (W/m)
q''	heat flux (W/m^2)
Re	Reynolds Number (dimensionless)
t	temperature ($^{\circ}\text{C}$)
T	absolute temperature (K)
U	overall heat transfer coefficient ($\text{W/m}^2\text{K}$)
v	velocity (m/s)
w	mass flow rate (kg/s)
β	coefficient of thermal expansion (K^{-1})
μ	dynamic viscosity (Pa s)
ν	kinematic viscosity (m^2/s)
ρ	mass density (kg/m^3)
Θ	temperature difference (K)

Chapter 10, Drying

10.1 Moisture Content

The moisture content of a material may be expressed in either of two different ways: (1) the wet-basis moisture content or (2) the dry-basis moisture content. The wet-basis moisture content is given by the following equation:

$$m = W_m / (W_m + W_d) \quad (10.1)$$

where

m = wet basis moisture content expressed as a decimal

W_m = mass of moisture (kg)

W_d = mass of dry matter (kg)

The dry-basis moisture content is given by:

$$M = W_m / W_d \quad (10.2)$$

where M = dry-basis moisture content expressed as a decimal.

The wet-basis and dry-basis moisture contents are thus related by the following equations:

$$m = M / (1 + M) \quad (10.3)$$

$$M = m / (1 - m) \quad (10.4)$$

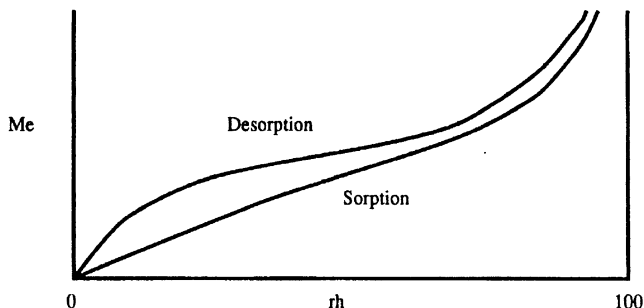


Figure 10.1. Typical equilibrium moisture content isotherms.

10.3 Equilibrium Moisture Content

Moisture is exchanged between a material and its surroundings until the material reaches some equilibrium moisture content at which there is no net gain or loss of moisture. The equilibrium moisture content is known to be a function of temperature, relative humidity, the physical properties of the material, and the previous moisture history of the material. A plot of the equilibrium moisture content versus relative humidity at a constant temperature is called an equilibrium moisture content isotherm. A sorption isotherm indicates that the equilibrium was reached in a wetting environment, and a desorption isotherm indicates that the equilibrium was reached in a drying environment.

Equilibrium moisture content isotherms for most materials are sigmoidal in shape (figure 10.1) with the desorption isotherm being above the sorption isotherm. Numerous theories have been suggested for describing the equilibrium moisture content isotherms.

10.3.1 Langmuir Equation

The Langmuir Equation (Langmuir, 1918) for describing adsorption of vapors was developed under the assumption of a monomolecular layer of the adsorbed gas developing on the surface of the material. The equation is:

$$v = v_m bp / (1 + bp) \quad (10.5)$$

where

v = volume of gas adsorbed (m^3)

v_m = volume of a unimolecular layer of the adsorbate (m^3)

p = vapor pressure of the gas (Pa)

b = parameter dependent upon the material and the temperature (Pa^{-1})

This equation gives an isotherm of the type shown in figure 10.2 for the adsorption of water vapor. Thus, the moisture content approaches a maximum value asymptotically as the relative humidity approaches 100%. This type of equation describes the adsorption of some gases by some inorganic materials but does not correctly describe the sorption of water vapor by biological materials.

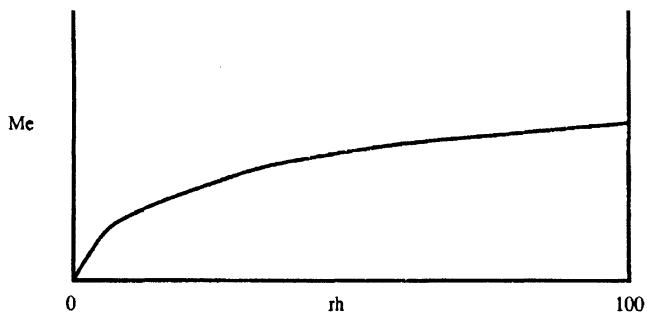


Figure 10.2. Langmuir equilibrium moisture content equation.

10.3.2 BET Equation

The BET Equation (named for Brunauer, Emmett, and Teller, 1938) was developed under the assumption of multimolecular layers of adsorbed gas on the surface of the adsorbent. The equation is:

$$v = cv_m p / \{(p_{\text{sat}} - p)[1 + (c - 1)(p/p_{\text{sat}})]\} \quad (10.6)$$

where

- v = volume of gas adsorbed (m^3)
- v_m = volume of a unimolecular layer of the gas (m^3)
- p = vapor pressure of gas (Pa)
- p_{sat} = saturation vapor pressure at the same temperature (Pa)
- c = parameter dependent upon the adsorbent material and the temperature (dimensionless)

The BET equation has been widely used to describe the adsorption of gases by inorganic materials and is used to determine the surface area of porous materials such as silica gel. A plot of the BET equation gives an isotherm such as the one shown in figure 10.3. Thus, a sigmoidal isotherm is obtained which is of the same general shape found experimentally for water vapor adsorption by biological materials. However, the

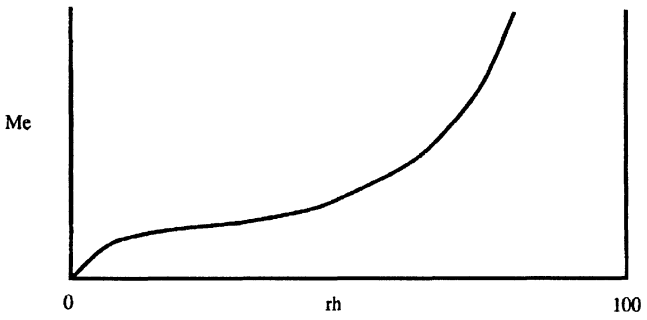


Figure 10.3. BET equilibrium moisture content equation.

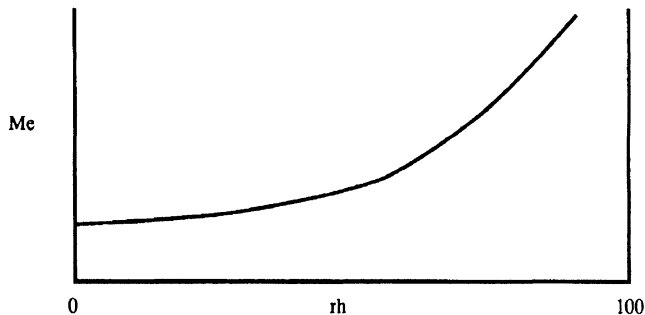


Figure 10.4. Smith equilibrium moisture content equation.

BET equation approaches infinite moisture contents too rapidly at relative humidities above approximately 50%.

10.3.3 Smith Equation

The Smith Equation (Smith, 1947) was developed specifically for the case of water vapor sorption at high relative humidities. The equation is:

$$v = v_b - v_m \ln(1 - rh) \quad (10.7)$$

where

v = volume of water adsorbed (m^3)

v_b = volume of bound water adsorbed (m^3)

v_m = volume of unimolecular layer of normally condensed moisture (m^3)

rh = relative humidity (expressed as a decimal)

This equation gives an isotherm of the form shown in figure 10.4. The equation has been found to fit experimental data quite accurately at high relative humidities. However, it could not be used at low relative humidities.

10.3.4 Henderson Equation

The best known and most widely used equation for the equilibrium moisture content of biological materials is an empirical equation developed by Henderson (1952):

$$1 - rh = \exp[-cT M_e^n] \quad (10.8)$$

where

rh = relative humidity (decimal)

T = absolute temperature (K)

M_e = equilibrium moisture content, dry-basis (percent)

c, n = empirical constants given for some materials in table 10-1

Although this relationship must be considered empirical, it has been found quite useful in describing the equilibrium moisture isotherms of many biological materials (especially if the c and n values are determined for the temperature of interest). Figure 10.5 gives experimental desorption curves for several of the materials listed in table 10-1.

Table 10-1. Values of equilibrium constants c and n for some materials

Material	c	n
Shelled corn	1.98×10^{-5}	1.90
Wheat	10.06×10^{-7}	3.03
Sorghum	6.12×10^{-6}	2.31
Soybeans	5.78×10^{-5}	1.52
Flaxseed	12.40×10^{-6}	2.02
Raisins	12.83×10^{-5}	1.02
Dried peaches	7.40×10^{-4}	0.564
Dried prunes	2.25×10^{-4}	0.865
Cotton	8.84×10^{-5}	1.70
Wood	9.61×10^{-5}	1.41
Spray-dried eggs	5.31×10^{-5}	2.00
Natural clay	13.55×10^{-5}	1.72

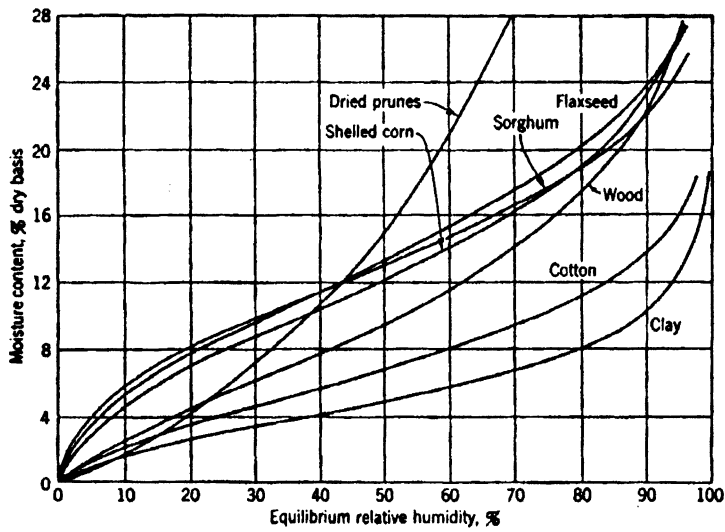


Figure 10.5. Equilibrium moisture curves of a number of materials at room temperature, approximately 25 C.

10.3.5 Young Equations

Equations have been developed (Young, 1966) based on the theory of moisture occurring as either (1) bound, (2) free or normally condensed, or (3) absorbed moisture. Equations for each type of moisture were developed and the sum of these equations gives the total moisture content of the material. The equation for the bound moisture is:

$$\theta = rh/[rh + (1 - rh)E] \quad (10.9)$$

where

θ = fraction of surface covered by a layer of bound water molecules (decimal)

rh = relative humidity (decimal)

$E = \exp[-(q_1 - q_2)/kT]$

q_1 = heat of adsorption of the bound layer of water molecules (J/molecule)

q_2 = normal heat of vaporization of water molecules (J/molecule)

k = Boltzmann's constant (1.38×10^{-23} J/molecule K)

T = absolute temperature (K)

The equation for the "free" or normally condensed moisture is:

$$\alpha = -Erh/[E - (E - 1)rh] + [E^2/(E - 1)] \ln\{[E - (E - 1)rh]/E\} \\ - (E + 1) \ln(1 - rh) \quad (10.10)$$

where α = total amount of normally condensed moisture measured in molecular layers.

The equation for the absorbed moisture during a sorption process is:

$$\gamma_s = \rho V \phi \quad (10.11)$$

where

γ_s = amount of absorbed moisture in sorption process (kg)

ρ = density of water (kg/m^3)

V = amount of absorbed moisture at saturation (m^3)

$\phi = rh \theta$ = fraction of surface covered by at least one layer of normally condensed moisture (decimal)

The equation for the absorbed moisture during a desorption process is:

$$\gamma_d = \rho V \theta rh_{\max} \quad (10.12)$$

where

γ_d = amount of absorbed moisture in a desorption process (kg)

rh_{\max} = maximum relative humidity with which the material has been in equilibrium prior to desorption process

Then the total dry-basis moisture contents for sorption and desorption processes are given by:

$$M_s = (\rho v_m / W_d)(\theta + \alpha) + (\rho V / W_d)\phi \quad (10.13)$$

and

$$M_d = (\rho v_m / W_d)(\theta + \alpha) + (\rho V / W_d)\theta r h_{\max} \quad (10.14)$$

where

M_s = total dry-basis moisture content for a sorption process (decimal)

M_d = total dry-basis moisture content for a desorption process (decimal)

v_m = volume of moisture in a unimolecular layer of water molecules on the surface of the cells (m^3)

W_d = mass of dry material (kg)

ρ = density of water (kg/m^3)

These equations give equilibrium moisture content isotherms similar to those shown in figure 10.5. The equations approximate the BET equation at low relative humidities and the Smith equation at high relative humidities. This combination has been found to quite accurately describe experimental isotherms. The equations have the unique feature that they can describe either sorption or desorption isotherms. None of the previously discussed theories distinguish between sorption and desorption. The constants of the equations would have to be determined from material having a similar moisture history. Different values of the constants would be determined for sorption processes than for desorption processes.

Table 10-2 gives the temperatures and drying times specified by ASAE for a number of grains.

Table 10-2. Oven temperature and heating period for moisture content determinations

Seed	Oven Temperature, ±1 C	Heating Time	
		hr	min
Alfalfa	130	2	30
Barley	130	20	0
Beans, edible	103	72	0
Bentgrass	130	1	0
Bluegrass	130	1	0
Bluestem	100	1	0
Bromegrass	130	0	50
Cabbage	130	4	0
Carrott	100	0	40
Clover	130	2	30
Collard	130	4	0
Corn	103	72	0
Fescue	130	3	0
Flax	103	4	0
Kale	130	4	0
Mustard	130	4	0
Oats	130	22	0
Onion	130	0	50
Orchardgrass	130	1	0
Parsley	100	2	0
Parsnip	100	1	0
Peanuts	130	6	0
Radish	130	1	10
Rape	130	4	0
Rye	130	16	0
Ryegrass	130	3	0
Safflower	130	1	0
Sorghum	130	18	0
Soybeans	103	72	0
Sunflower	130	3	0
Timothy	130	1	40
Turnip	130	4	0
Wheat	130	19	0

Table 10-3. Relationship between vapor pressure ratio (i.e., relative humidity), height of water column, and pF values

p/p_{sat} (%)	Height of H ₂ O column (cm)	pF
0.08	10^7	7
49.0	10^6	6
93.0	10^5	5
99.3	10^4	4
99.9	10^3	3
99.99	10^2	2
99.999	10	1
100.0	0	—

support. The relationship between the column height and the relative vapor pressure is:

$$h = -(RT/Mg) \ln(p/p_{\text{sat}}) \tag{10.15}$$

where

- h = height of column of water in capillary (m)
- R = universal gas constant (8.314×10^3 Pa m³/kg-mole K)
- T = absolute temperature (K)
- M = molecular mass of water (kg/kg-mole)
- g = gravitational acceleration (m/s²)
- p = vapor pressure over the meniscus (Pa)
- p_{sat} = saturation vapor pressure for flat water surface (Pa)

The value calculated from this equation is commonly referred to as the suction. As a convenience, units of suction are sometimes expressed as a pF value defined by the following relationship:

$$pF = \log(100 h) = \log[-(100RT/Mg) \ln(p/p_{\text{sat}})] \tag{10.16}$$

The relationship between vapor pressure ratio (i.e. relative humidity), height of water column, and the pF are given in table 10-3.

Table 10-4 gives a brief summary of the advantages and disadvantages of each of the previously discussed moisture measurement techniques as given by Young (1991).

Table 10-4. Advantages and disadvantages of various moisture measuring methods

1. Oven Methods	a. Simple and direct.	a. Heat may cause decomposition of dry matter creating water not originally present. b. Other volatile materials may be driven off. c. Degree of grinding and time, temperature, and pressure may affect results. d. Moisture may be gained or lost during grinding.
2. Desiccant drying	a. Heat not required for drying.	a. Lengthy time requirements. b. Moisture may be lost or gained during grinding.
3. Distillation methods	a. Water may be determined directly rather than as a weight loss.	a. Other volatile substances which are soluble in water may cause errors. b. Distilling time to obtain complete distillation of water is uncertain. c. Moisture may be lost or gained during grinding. d. Sample is destroyed.
4. Chemical methods	a. Chemical reactions based on water present should theoretically produce high accuracy.	a. Water may not all take part in the reaction. b. Moisture may be lost or gained during grinding. c. Sample is destroyed.
5. Gas chromatography	a. Highly accurate determination of water in methanol extract.	a. Complete extraction of water is uncertain. b. Expensive equipment required. c. Moisture may be lost or gained during grinding. d. Sample is destroyed.
6. Electrical methods	a. Rapid, simple, and may be non-destructive	a. Variations in electrical properties of the dry matter may cause errors. b. Moisture distribution may affect readings. c. Ratio of "free" to "bound" water must be constant at a given moisture content.
7. Spectrophotometric methods	a. Measurement based on fundamental properties of the water molecule. b. May be nondestructive in some cases.	a. Water in different forms may give different results. b. Dry matter absorbs some energy and presents some calibration difficulty. c. Moisture distribution affects the readings. d. Expensive equipment required in some cases.

Table 10-4. (Continued)

8. Nuclear methods	a. Rapid.	a. Senses total hydrogen rather than water.
	b. High accuracy for properly calibrated NMR instruments.	b. Different energy forms of water may cause problems in separation of water signals from nonwater hydrogen NMR signals.
	c. Nondestructive.	c. Other elements may affect neutron scattering readings.
9. Equilibrium relative humidity methods	a. Relatively simple.	d. Expensive equipment required.
	b. Nondestructive.	a. Relationship between moisture content and equilibrium relative humidity must be known.
		b. Hysteresis in material causes variations from calibration curves.
		c. Inaccurate at high relative humidities.
		d. Material must be in a state of equilibrium with its environment.
10. Suction methods		e. Relative humidity measurement is difficult.
	a. Relatively simple.	a. Relationship between moisture content and suction must be known.
	b. Nondestructive.	b. Hysteresis in material causes variations from calibration curve.
	c. Increased accuracy at very high moisture levels.	c. Cannot be used at low moisture levels.

The energy required for drying may be provided by radiation, conduction, or convection. The convection case is represented by the adiabatic evaporation of moisture from the surface of a wet-bulb thermometer. The equation representing the drying is:

$$dW/d\tau = f_v A(p_{\text{sat}} - p_v) = f A(t_a - t_s)/h_{fg} \quad (10.17)$$

where

$dW/d\tau$ = drying rate (kg/s)

- f = convective heat transfer coefficient at the water-air interface ($\text{W/m}^2\text{K}$)
 A = water surface area (m^2)
 h_{fg} = latent heat of water at water surface temperature (J/kg)
 t_a = air temperature (K)
 t_s = water surface temperature (K)
 f_v = water vapor transfer coefficient at the water-air interface ($\text{kg/s m}^2 \text{ Pa}$)
 p_{sat} = saturation water vapor pressure at t_s (Pa)
 p_v = water vapor pressure in the air (Pa)

Values of f_v and f were determined by Gamson et al. (1943) for drying by forcing air through beds of moist spherical or cylindrical pellets. For vapor transfer, with Reynolds number greater than 350,

$$(f_v D_p p_{\text{atm}} / D_v) = 0.989 (D_p G / \mu)^{0.59} (\mu / D_v \rho)^{1/3} \quad (10.18)$$

and for heat transfer:

$$(f D_p / k) = 1.064 (D_p G / \mu)^{0.59} (c_p \mu / k)^{1/3} \quad (10.19)$$

where

- D_p = diameter of particle (m)
 D_v = diffusivity of water vapor (m^2/s)
 k = thermal conductivity of air (W/mK)
 G = mass velocity (kg/s m^2)
 ρ = density of air (kg/m^3)
 μ = viscosity of air (kg/s m)
 c_p = specific heat of air (J/kg K)
 p_{atm} = total pressure of the atmosphere (Pa)

Note that:

$$(D_p G / \mu) = Re = \text{Reynolds number} \quad (10.20)$$

$$(c_p \mu / k) = Pr = \text{Prandtl number} \quad (10.21)$$

$$(\mu / D_v \rho) = Sm = \text{Schmidt number} \quad (10.22)$$

If the energy for the evaporation of moisture were supplied by radiation rather than by convection the following equation would describe the process:

$$dW/d\tau = f_v A (p_{\text{sat}} - p_v) = \varepsilon A \sigma (T_w^4 - T_s^4) / h_{fg} \quad (10.23)$$

where

- ε = emissivity of the water surface (dimensionless)
 σ = Stefan-Boltzmann constant ($5.73 \times 10^{-8} \text{ W/m}^2\text{K}^4$)
 T_w = absolute temperature of radiating walls (K)
 T_s = absolute temperature of water surface (K)

10.5.2 Falling Rate Period

Practically all agricultural drying takes place in the falling-rate period. The falling-rate period is bounded by equilibrium moisture contents of an equilibrium moisture curve between zero and nearly 100% relative humidity. Moisture contents near the 100% level would approximate a constant rate drying period because as moisture evaporates the equilibrium relative humidity and thus the vapor pressure driving force change only very slightly.

Drying in the falling-rate period involves two processes: (1) the movement of moisture within the material to the surface and (2) removal of the moisture from the surface.

10.5.2.1 Sherwood's Exponential Drying Equation

An equation which has been most used for describing the drying of agricultural products was developed by Sherwood (1936). He began with the assumption that the drying rate was proportional to the difference in moisture content between the material being dried and the equilibrium moisture content at the drying air state or:

$$dM/d\tau = -k(M - M_e) \quad (10.24)$$

where

$dM/d\tau$ = drying rate ($\text{kg}_{\text{H}_2\text{O}}/\text{kg}_{\text{dm}} \text{ h}$)

k = drying parameter (h^{-1})

M = moisture content of material, $\text{kg}_{\text{H}_2\text{O}}/\text{kg}_{\text{dm}}$

M_e = equilibrium moisture content of material at conditions of drying air ($\text{kg}_{\text{H}_2\text{O}}/\text{kg}_{\text{dm}}$)

Integration of equation 10.24 and substitution of the boundary condition that $M = M_o$ at $\tau = 0$ yields:

$$(M - M_e)/(M_o - M_e) = \exp(-k\tau) \quad (10.25)$$

The exponential drying curve is analogous to Newton's Law of heating or cooling such that the internal resistance to moisture movement is considered to be negligible with the surface resistance controlling the drying process.

Also, if we accept the theory that the vapor pressure gradient is the driving force for moisture movement,

$$dM/d\tau = -k_1(p_{\text{sat}} - p_v) \quad (10.26)$$

In order for equation 10.26 to take the form of 10.24, the following relations must hold:

$$k(M - M_e) = k_1(p_{\text{sat}} - p_v) \quad (10.27)$$

or in general the equilibrium moisture content must be given by:

$$M_e = (k_1/k)p + c_1 \quad (10.28)$$

Thus, equation 10.25 is based on the assumptions that the equilibrium moisture content is a linear function of vapor pressure and that the limiting resistance to moisture flow is at the surface. The assumption of a linear relationship between equilibrium moisture

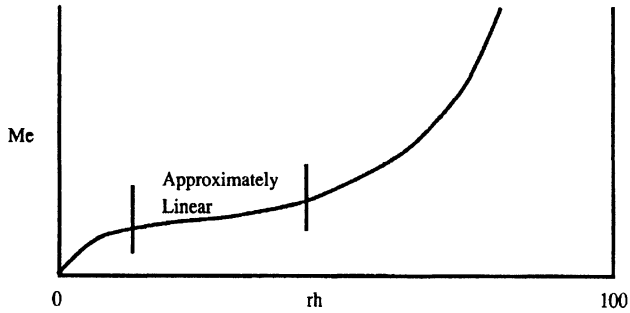


Figure 10.6. Assumption of linear relationship between equilibrium moisture content and vapor pressure at medium relative humidities.

content and vapor pressure is a fairly good approximation at medium relative humidities (fig. 10.6). However, the assumption of negligible internal resistance is questionable in most cases. The equation based on these assumptions has been found quite useful, however, in describing many drying processes.

10.5.2.2 Diffusion Equations

If the surface resistance to moisture transfer is negligible in comparison to the internal resistance, diffusion equations may be used to describe the drying process. These equations are based on a special case of Fick's first law of diffusion which states that:

$$J_x = -D(\partial C / \partial x) \quad (10.29)$$

where

- J_x = flux of the diffusing medium, in this case water (kg/sm^2)
- D = mass diffusivity of water (m^2/s)
- C = concentration of water (kg/m^3)
- x = distance (m)

This equation is completely analogous to the Fourier-Biot Law for heat conduction. Equation 10.29 states that the rate of movement of water within the material is proportional to the concentration gradient. However, the question arises as to whether the water moves as a liquid or as a vapor. If the movement is in the liquid form, then the concentration gradient to be used in equation 10.29 is the liquid concentration gradient. If the movement is in the vapor state, then the vapor concentration gradient should be used. It is also possible that water might be transferred by both liquid and vapor diffusion in which case the total flux of water would have to be the sum of the liquid flux and the vapor flux. Most drying work to date has assumed either liquid or vapor diffusion but not both. We shall consider these two cases.

10.5.2.2.1 Liquid Diffusion. In the case of liquid diffusion we have:

$$J_x = -D_{\text{liq}}(\partial C_{\text{liq}} / \partial x) \quad (10.30)$$

where

C_{liq} = liquid concentration (kg/m^3)

D_{liq} = mass diffusivity of liquid water (m^2/s)

In a manner similar to the derivation of the Fourier-Poisson equation for heat conduction, Fick's 2nd Law of diffusion can be developed:

$$(\partial C_{\text{liq}}/\partial \tau) = D_{\text{liq}}((\partial^2 C_{\text{liq}}/\partial x^2) + (\partial^2 C_{\text{liq}}/\partial y^2) + (\partial^2 C_{\text{liq}}/\partial z^2)) \quad (10.31)$$

Since,

$$M = C_{\text{liq}}/\rho_{\text{dm}} \quad (10.32)$$

where

M = moisture content (dry basis)

ρ_{dm} = density of the dry matter (kg/m^3)

equation 10.31 may be written as:

$$\partial M/\partial \tau = D_{\text{liq}}((\partial^2 M/\partial x^2) + (\partial^2 M/\partial y^2) + (\partial^2 M/\partial z^2)) \quad (10.33)$$

where $\partial M/\partial \tau$ = rate of change of moisture content with respect to time, ($\text{kg}_{\text{H}_2\text{O}}/\text{kg}_{\text{dm}} \text{ s}$).

If a plane sheet of material at an initial moisture content, M_o , is placed in an environment for which its equilibrium moisture content is M_e , the solution to equation 10.33 is:

$$\begin{aligned} (M - M_o)/(M_e - M_o) &= 1 - (4/\pi) \sum_{n=0}^{\infty} ((-1)^n/(2n+1)) \\ &\exp(-D_{\text{liq}}(2n+1)^2\pi^2\tau/(4L^2)) \cos((2n+1)\pi x/(2L)) \end{aligned} \quad (10.34)$$

where

M = moisture content, dry basis (at time τ and point x)

L = half thickness of sheet (m)

The average moisture content at any time is given by:

$$(\bar{M} - M_e)/(M_o - M_e) = \sum_{n=0}^{\infty} \{8/[(2n+1)^2\pi^2]\} \exp(-D_{\text{liq}}(2n+1)^2\pi^2\tau/(4L^2)) \quad (10.35)$$

where \bar{M} = average moisture content, dry basis (at time τ). The analogous solutions for a sphere are:

$$\begin{aligned} (M - M_o)/(M_e - M_o) &= 1 + (2a/\pi r) \sum_{n=1}^{\infty} ((-1)^n/n) \sin(n\pi r/a) \\ &\exp(-D_{\text{liq}}n^2\pi^2\tau/a^2) \end{aligned} \quad (10.36)$$

$$(\bar{M} - M_e)/(M_o - M_e) = (6/\pi^2) \sum_{n=0}^{\infty} (1/n^2) \exp(-D_{\text{liq}}n^2\pi^2\tau/a^2) \quad (10.37)$$

where

a = radius of the sphere (m)

r = radial distance from the center (m)

These equations assume a constant diffusivity, uniform initial concentrations, and constant surface concentrations. Under a few specialized cases, the differential equation can be solved for cases of variable diffusivities. Non-uniform initial moisture contents may also be analyzed.

For cases where the surface resistance to moisture movement is not negligible, the equations have also been solved considering the finite value of the surface conductance.

10.5.2.2.2 Vapor Diffusion. In the case of vapor diffusion we have:

$$J_x = -D_v(\partial C_v/\partial x) \quad (10.38)$$

where

C_v = vapor concentration (kg/m³)

D_v = vapor mass diffusivity (m²/s)

The diffusion of the vapor must take place through the pores of the solid material and the liquid water present within the material provides a source of vapor due to evaporation within the body. The differential equation describing the vapor diffusion process is for a constant diffusivity:

$$D_v((\partial^2 C_v/\partial x^2) + (\partial^2 C_v/\partial y^2) + (\partial^2 C_v/\partial z^2)) = v(\partial C_v/\partial \tau) + (1 - v)\rho_s(\partial M/\partial \tau) \quad (10.39)$$

where

v = void fraction or porosity of body (dimensionless)

ρ_s = density of solid material (kg/m³)

τ = time (s)

M = moisture content, dry basis (decimal)

Equation 10.39 contains two dependent variables, c_v and M . Therefore, the relationship between the two must be known or assumed before the equation can be solved. Most attempts to solve equation 10.39 have begun with the assumption that the equilibrium moisture content is a linear function of the vapor concentration and temperature.

$$M = \alpha + \beta C_v - \gamma T \quad (10.40)$$

where

T = absolute temperature (K)

α, β, γ = constants having appropriate dimensions for dimensional homogeneity

Then for a constant temperature:

$$\partial M/\partial \tau = \beta(\partial C_v/\partial \tau) \quad (10.41)$$

Substituting equation 10.41 into 10.40 gives:

$$\partial C_v / \partial \tau = (D_v / (v + (1 - v)\rho_s \beta)) ((\partial^2 C_v / \partial x^2) + (\partial^2 C_v / \partial y^2) + (\partial^2 C_v / \partial z^2)) \quad (10.42)$$

Equation 10.42 is analogous to equation 10.31 for liquid diffusion with the $(v + (1 - v)\rho_s \beta)$ term being the primary difference. Again, if the moisture content is linearly related to the vapor concentration then:

$$\partial M / \partial \tau = (D_v / (v + (1 - v)\rho_s \beta)) ((\partial^2 M / \partial x^2) + (\partial^2 M / \partial y^2) + (\partial^2 M / \partial z^2)) \quad (10.43)$$

The only difference between equation 10.43 for vapor diffusion and equation 10.33 for liquid diffusion is the diffusivity function if the linear relationship between vapor concentration and moisture content is assumed. For equivalent drying relationships, the following identity must hold:

$$D_{\text{liq}} = D_v / (v + (1 - v)\rho_s \beta) \quad (10.44)$$

Thus, the assumption of liquid or vapor diffusion only affects the choice of the diffusivity to be used if the moisture content is assumed to vary linearly with vapor concentration and the material being dried is homogenous. However, the vapor diffusion equation is more appropriate for use with composite bodies composed of materials having different physical and hygroscopic properties.

Also, with the assumption of a vapor diffusion model, the heat transfer equation may be solved simultaneously for the case where the temperature is variable. The heat transfer equation for this case is:

$$\begin{aligned} k((\partial^2 T / \partial x^2) + (\partial^2 T / \partial y^2) + (\partial^2 T / \partial z^2)) = & c_s(1 - v)\rho_s(\partial T / \partial \tau) \\ & + c_w(1 - v)\rho_s M(\partial T / \partial \tau) - h_{fg}(1 - v)\rho_s(\partial M / \partial \tau) - q''' \end{aligned} \quad (10.45)$$

where

- k = thermal conductivity of the body (W/mK)
- T = temperature (K)
- c_s = specific heat of solid (J/kg K)
- c_w = specific heat of water (J/kg K)
- h_{fg} = latent heat of vaporization of the moisture (J/kg)
- q''' = rate of internal heat generation (W/m³)

By a numerical solution of the vapor diffusion equation and the heat transfer equation simultaneously it has been shown that the heat transfer into the body is only a limiting factor if:

$$Le_m < 60 \quad (10.46)$$

where

$$\begin{aligned} Le_m &= k(v + (1 - v)\rho_s \beta) / (D_v(1 - v)\rho_s(c_s + c_w M + h_{fg}\gamma)) \\ Le_m &= \text{modified Lewis Number (dimensionless)} \end{aligned} \quad (10.47)$$

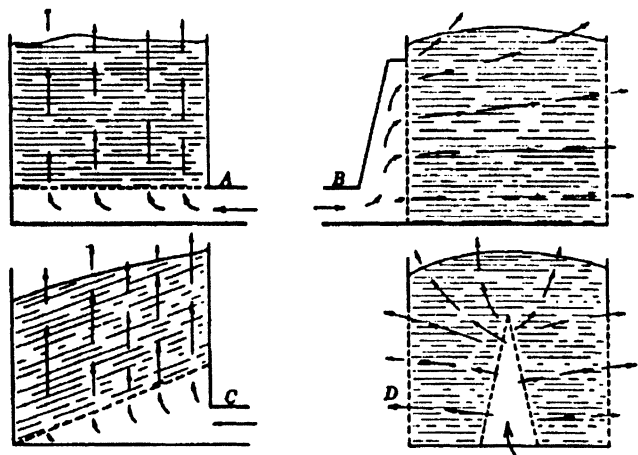


Figure 10.17. Some deep-bed drying arrangements.

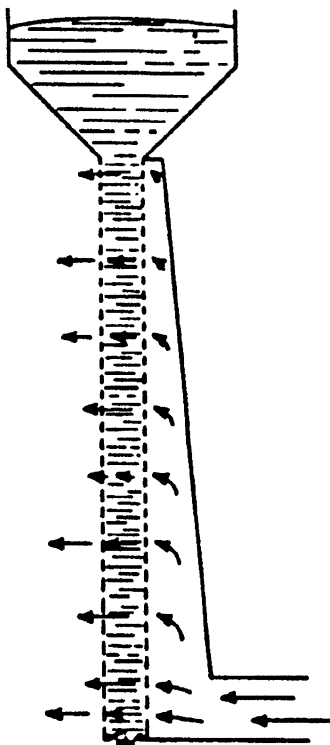


Figure 10.18. A cross-flow drier.

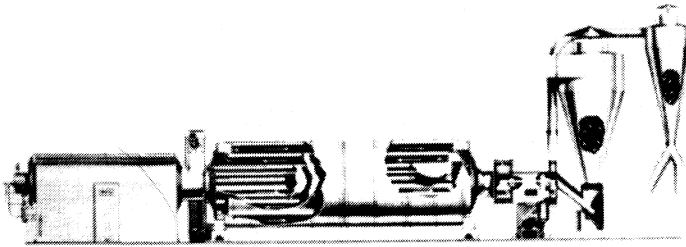


Figure 10.19. A multiple-drum drier with a direct-fired heater and cyclones for collecting and cooling the dried product. (Courtesy of The Heil Co.)

The state of the exhaust air necessary for a rotary drier may be determined from the following moisture balance:

$$(Q/v)(H_e - H_i) = \dot{m}_{dm}(M_i - M_f) \quad (10.99)$$

where

Q = volumetric flow rate of air through drier (m^3/s)

v = specific volume of entering air (m^3/kg)

H_e = humidity ratio of air at exhaust (kg/kg)

H_i = humidity ratio of entering air (kg/kg)

\dot{m}_{dm} = flow rate of dry matter (kg/s)

M_i = entering moisture content of material, dry basis (decimal)

M_f = exiting moisture content of material, dry basis (decimal)

The volume of the drum and thus the retention time necessary to achieve a specified amount of drying will be determined by the drying characteristics of the material being dried.



Figure 10.20. Cars of trays being moved into a tunnel drier. Note the tracks used to guide the cars. (Courtesy of California Prune and Apricot Growers Association.)

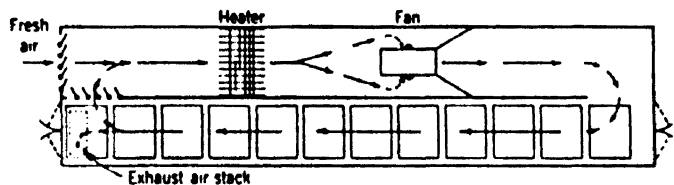


Figure 10.21. Plan of a tunnel drier. The cars move from left to right for counterflow operation, from right to left for parallel flow.

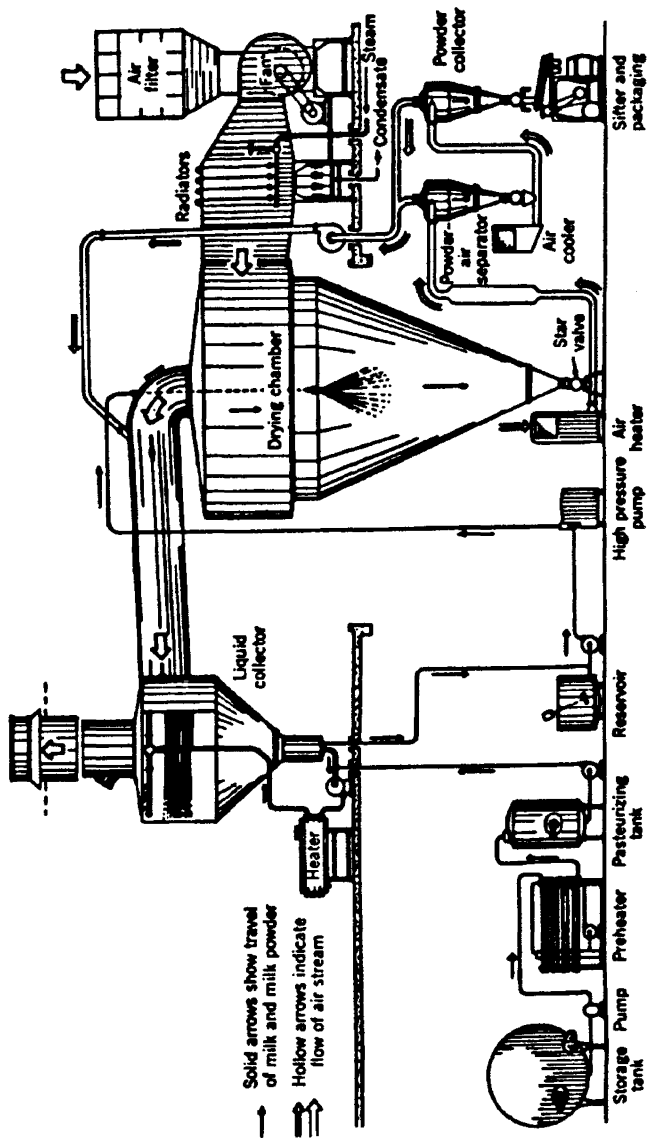


Figure 10.22. A continuous-flow spray drier for milk products. (Courtesy of Swenson Evaporator Co.)

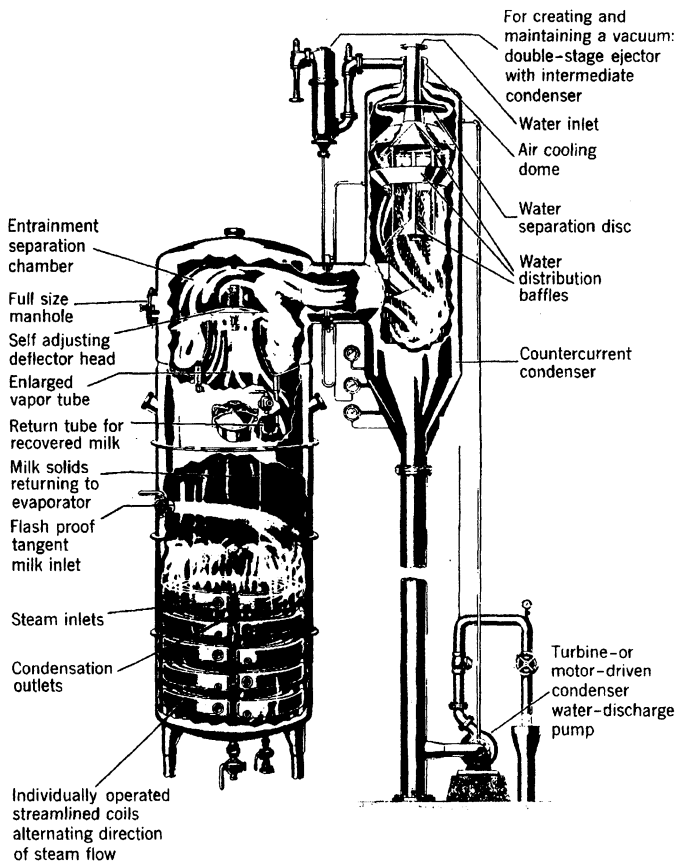


Figure 10.23. Vacuum pan with entrainment separation chamber and countercurrent condenser used for milk concentration. (Courtesy of Arthur Harris and Co.)

Consider an evaporator set up to concentrate milk on a constant flow basis. The following equations describe the energy balances in the system. In the evaporating chamber:

$$\dot{m}_s h_g + \dot{m}_m c_m t_i = \dot{m}_s h_f + \dot{m}_v h_v + (\dot{m}_m - \dot{m}_v) c_m t_v + \text{losses} \quad (10.100)$$

where

\dot{m}_s = mass flow rate of steam (kg/s)

\dot{m}_m = mass flow rate of milk (kg/s)

\dot{m}_v = vaporization rate of water from milk = condensation rate of water in condensing chamber (kg/s)

h_g = enthalpy of entering steam (kJ/kg)

h_f = enthalpy of steam condensate (kJ/kg)

h_v = enthalpy of evaporating water at temperature t_v (kJ/kg)

c_m = specific heat of entering milk (kJ/kgK)

t_i = entering temperature of milk (K)

t_v = temperature of vaporization of milk (K)

Equation 10.100 can be rearranged to obtain:

$$\dot{m}_s (h_g - h_f) + \dot{m}_m c_m (t_i - t_v) = \dot{m}_v h_{fg} + \text{losses} \quad (10.101)$$

where h_{fg} = heat of vaporization of water at t_v (kJ/kg).

The following equation gives an energy balance for the steam coils in the evaporating chamber:

$$AU(t_s - t_v) = \dot{m}_s (h_g - h_f) \quad (10.102)$$

where

A = effective steam coil area (m²)

U = overall heat transfer coefficient of steam coils (kW/m²K)

t_s = effective average temperature of steam (K)

An energy balance in the condensing chamber results in:

$$\dot{m}_v h_v + \dot{m}_w h_{f1} = (\dot{m}_w + \dot{m}_v) h_{f2} \quad (10.103)$$

where

\dot{m}_w = mass flow rate of condensing water (kg/s)

h_{f1} = enthalpy of entering condensing water (kJ/kg)

h_{f2} = enthalpy of condensed water and exiting condensing water (kJ/kg)

or

$$\dot{m}_v (h_v - h_{f2}) = \dot{m}_w (h_{f2} - h_{f1}) = \dot{m}_w c_w (t_2 - t_1) \quad (10.104)$$

where

c_w = specific heat of liquid water (kJ/kgK)

t_1 = entrance temperature of condensing water (K)

t_2 = exit temperature of condensing water (K)

When the condenser is counterflow, as in figure 10.22, $t_v - t_2$ is usually 2 to 3 C. But if the unit is designed for parallel flow performance, the difference will probably be approximately 8 C.

When operating factor values are considered for a specific “pan”, the boiling temperature, t_v , is controlled by the condensing water rate, \dot{m}_w .

The steam coils are usually fed with steam at 35 to 70 kPa. The boiling is extremely vigorous because of the high temperature difference. Thus, a high heat rate is possible with minimum heat exchange surface and a “cooked” flavor is improbable owing to the surface speeds of the liquid.

This unit can operate on a batch basis or continuously by means of a suitable pump that continuously removes liquid from the bottom of the “pan” and by continuous feeding in of the liquid stock.

Nomenclature

a	radius of sphere (m)
A	surface area (m^2)
c_p	specific heat of air (J/kg K)
C	concentration (kg/m^3)
D	mass diffusivity (m^2/s)
	number of depth units (dimensionless)
D_p	diameter of particle (m)
D_{liq}	diffusivity of liquid water (m^2/s)
D_v	diffusivity of water vapor (m^2/s)
E	$\exp[-(q_1 - q_2)/kT]$
f	convective heat transfer coefficient ($\text{W/m}^2\text{K}$)
f_v	water vapor transfer coefficient (kg/s m^2)
g	gravitational acceleration (m/s^2)
G	mass velocity (kg/s m^2)
h	height of column of water in capillary (m)
h_{fg}	latent heat of vaporization of water (J/kg)
H	time of half response (h)
J	flux of diffusing medium (kg/s m^2)
k	Boltzmann's constant, 1.38×10^{-23} J/molecule K
	thermal conductivity (W/mK)
	drying parameter (h^{-1})
L	half thickness of sheet (m)
Le_m	modified Lewis Number (dimensionless)
m	moisture content (wet-basis decimal)
\dot{m}	mass flow rate (kg/s)
M	moisture content, dry-basis (decimal)
M_o	initial moisture content (dry basis)

M_d	equilibrium moisture content for a desorption process (dry basis)
M_e	equilibrium moisture content (dry basis)
M_s	equilibrium moisture content for a sorption process (dry basis)
p	vapor pressure of gas (Pa)
p_{atm}	total atmospheric pressure (Pa)
p_{sat}	saturation vapor pressure (Pa)
p_v	water vapor pressure in air (Pa)
Pr	Prandtl Number (dimensionless)
R	universal gas constant (8.314×10^3 Pa m ³ /kg-mole K)
Re	Reynolds Number (dimensionless)
q_1	heat of adsorption of bound layer of water molecules (J/molecule)
q_2	normal heat of vaporization of water molecules (J/molecule)
rh	relative humidity (decimal)
rh_{max}	maximum relative humidity with which the material has been in equilibrium prior to desorption process (decimal)
Sm	Schmidt Number (dimensionless)
t	temperature (C)
T	absolute temperature (K)
v	volume of gas adsorbed (m ³)
v_b	volume of bound water adsorbed (m ³)
v_m	volume of a unimolecular layer of the adsorbate (m ³)
V	amount of absorbed moisture at saturation (m ³)
W_m	mass of moisture (kg)
W_d	mass of dry matter (kg)
Y	number of time units (dimensionless)
α	total amount of normally condensed moisture measured in molecular layers
ε	emissivity of water surface (dimensionless)
ϕ	fraction of surface covered by at least one layer of normally condensed moisture (decimal)
γ_d	amount of absorbed moisture in a desorption process (kg)
γ_s	amount of absorbed moisture in a sorption process (kg)
μ	viscosity of air (kg/s m)
ν	porosity (dimensionless)
ρ	density (kg/m ³)
ρ_s	density of solid (kg/m ³)
σ	Stefan-Boltzmann constant (5.73×10^{-8} W/m ² K ⁴)
τ	time (s)
θ	fraction of surface covered by a layer of bound water molecules (decimal)