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Chapter 2

Reviewing the Basics

One objective of this book is to provide a subject matter area that can be used to increase the reader's ability to apply statics and the mechanics of deformable solids to design problems. This chapter contains a review of some topics to insure a common level of knowledge in these areas. Some of these topics are covered in all mechanics courses and should be familiar. Other topics are more applied and may not have been included in engineering science courses. The collection of topics is numeric and provides a group of practice problems preliminary to the start of the design calculations. The chapter starts with a discussion of the simple or average stresses and illustrates how these quantities are used to obtain bolt and pin diameters and to determine the thickness of connecting plates. This is followed by a discussion of systems of axial force members and the axial force diagram. About half of the chapter is spent on items related to beam design including the calculation of section properties and the construction of shear force and bending moment diagrams.

2.1 The Simple or Average Stresses

There is a group of stress calculations that use the generic equation

$$\text{stress} = \frac{\text{force}}{\text{area}} \tag{2.1}$$

The stress can be a normal stress, σ, or a shear stress, τ, depending on how the applied force acts on the area. Equation (2.1) is used because it gives the exact value of the stress or the stress distribution is so complex that all an engineer can do is calculate an average value and apply a factor of safety. The latter occurs in the design of bolted or riveted connections. Some people refer to the stress calculated using (2.1) as a simple stress because of the simplicity of the equation while others refer to the stress as an average value. Five different uses for (2.1) are discussed in this section. A brief review of why we need the calculation is followed in some cases by examples that illustrate how the calculations are used in a design situation.

2.1.1 The Normal Stress in Straight Bars

The first application of (2.1) is the calculation of the normal stress in straight bars subjected to axial tension or axial compression loads applied perpendicular to the area and at the centroid of the cross section (Figure 2.1). The resulting normal stress is uniform and its value is given by

$$\sigma = \frac{P}{A} \tag{2.2}$$

where σ is the normal stress, P is the applied load, and A is the cross-section area. The normal stress value in (2.2) is the only application of (2.1) that gives the correct stress value. Equation (2.2) is used in the design of axial tension members and axial compression members.

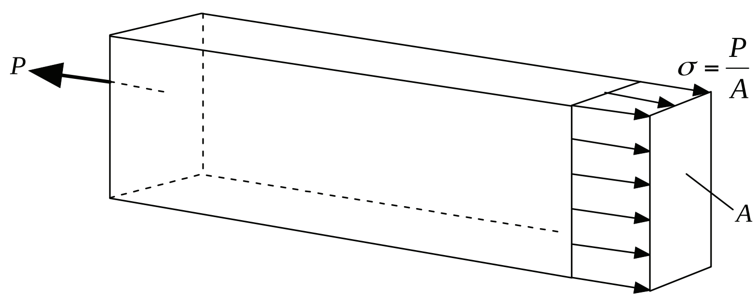


Figure 2.1 The normal stress in an axial tension member.

2.1.2 Bolts and Rivets in Shear

The need to connect members has existed for as long as individual components have been manufactured. For many years, the connection was made by riveting plates to the primary members (Figure 2.2). The design of bolted connections has been refined in recent years and on-site assembly of steel members is currently done using bolts. The shear stresses in bolted and riveted connections are calculated using the same equation so the words bolted and riveted are often interchanged. Riveted connections, however, are easier to draw so rivets are used in the diagrams.

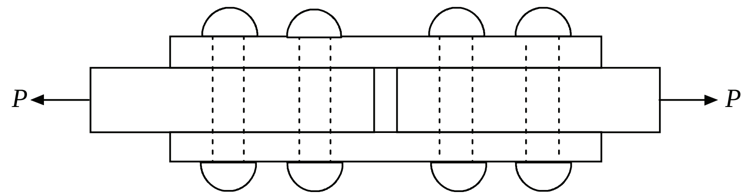


Figure 2.2 A tension joint with rivets in double shear.

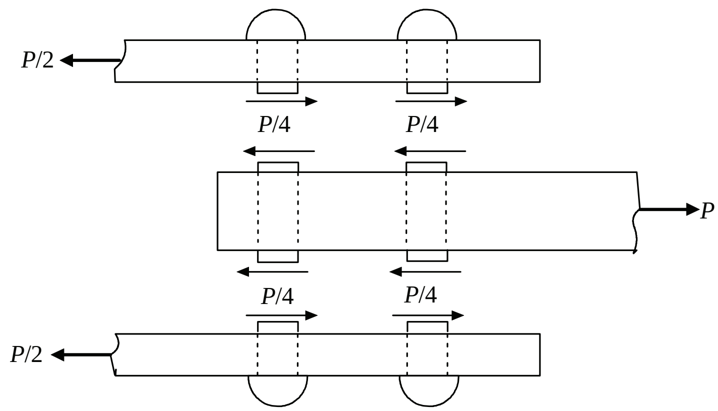


Figure 2.3 A free body diagram of a riveted joint in double shear.

A free body diagram of the riveted connection in Figure 2.2 is shown in Figure 2.3. The rivets are said to be in double shear because the axial force is transmitted through the joint using two shear planes. The transfer of the axial force through a joint is a very complicated problem and a complete mathematical analysis is probably impossible. Even experimental measurements are difficult. Because of the difficulty in solving the force transfer problem and the need to design bolted and riveted joints, engineers base the design on simple calculations and use a safety factor to ensure the design is adequate. The assumptions for a joint similar to Figure 2.2 are (1) each connecting plate transfers an equal part of the force and (2) each rivet in a connecting plate transfers an equal share of the loading. Each shear plane for a bolt or rivet is assumed to carry one-half of the load transferred by the fastener. The shear stress on one shear plane for one rivet is calculated using

$$\tau_{ave} = \frac{V}{A_v} \tag{2.3}$$

where τ_{ave} is the average shear stress, V is the shear force, and A_v is the shear area which is the cross-sectional area of the rivet. As an example, suppose the applied load on the joint in Figure 2.2 is 20 kips and the rivets have a cross-sectional area of 0.44 in^2 (0.750 inch diameter), the shear stress on each shear plane is

$$\tau_{ave} = \frac{(20\,\text{kips}/4)}{0.44\,\text{in}^2} = 11.4\,\text{ksi} \tag{2.4}$$

Since each shear plane transfers one-fourth of the load, the value of four in (2.4) can be split into the number of rivets multiplied by two shear areas per rivet. Some people prefer to write (2.3) in the form

$$\tau_{ave} = \frac{V}{2NA} \tag{2.5}$$

where N is the number of rivets and A is the cross-sectional area of one rivet. The application of (2.5) to the 20 kip loading gives

$$\tau_{ave} = \frac{20\,\text{kips}}{(2)(2)(0.44\,\text{in}^2)} = 11.4\,\text{ksi}$$

The evaluation of the shear stress in bolts or rivets using one of (2.4) or (2.5) is a matter of personal preference.

Equation (2.3) can be used in a design problem to determine the bolt diameter given the number of fasteners or to determine the number of fasteners given a diameter. An allowable shear stress value must be provided for each situation. The following example illustrates the determination of a bolt diameter.

Example 2.1 ■■■■

A tension connection similar to Figure 2.2 consisting of four bolts, two on each side of the joint, transfers a 32 kip loading. Determine the minimum bolt diameter when the allowable shear stress is τ_{all} = 21 ksi.

Solution:

There are two rivets with four shear planes on each side of the joint. Substitution into (2.3) gives

$$\tau_{ave} = \frac{V}{A_v} = \frac{(32\text{kips}/4)}{A_v} = \tau_{all} = 21\,\text{ksi}$$

which rearranges into

$$A_v = \frac{32\,\text{kips}}{4(21\,\text{ksi})} = 0.381\,\text{in}^2 = \frac{\pi d^2}{4}$$

or

$$d^2 = \frac{4(0.381\ \text{in}^2)}{\pi} = 0.485\,\text{in}^2$$

and

$$d = 0.697\ \text{in}$$

The minimum bolt diameter is 0.70 inch. A 0.750 inch diameter bolt is specified since this is the next available size with a diameter greater than 0.70 inch. Available bolt sizes are discussed in Chapter 6. ■■■■

2.1.3 A Single Pin in Double Shear

Many machines used in the construction industry have pin connections. Pins are removable and a pinned joint has only one fastener. This type of connection is used when (1) the machine has hydraulic cylinders, (2) when the machine must be disassembled for transportation, and (3) when the machine has multiple attachments to perform multiple functions. A simple pin-type connection is the clevis arrangement shown in Figure 2.4. The pin is often held in place by a OD snap ring which fits into a circular grove near each end.

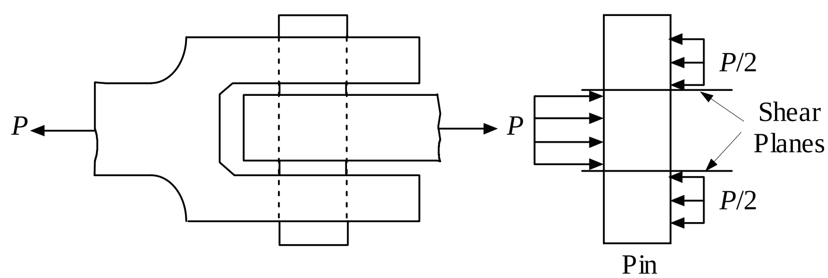


Figure 2.4 A clevis connection with a pin in double shear.

The primary objective when designing a pinned joint is to determine the pin diameter given an allowable shear stress. The pin experiences a pair of shear forces that result from the axial force transmitted through the joint. The evaluation of the shear stress is an application of (2.3)

$$\tau = \frac{V}{A_v} = \frac{(P/2)}{\left(\dfrac{\pi\, d_p^2}{4}\right)} = \frac{2\,P}{\pi\, d_p^2} \tag{2.6}$$

A commonly accepted value for the allowable shear stress in circular bars is $\tau_{all} = 0.40\ F_y$ where F_y is the yield stress for the steel used to make the pin. Substituting $0.40\ F_y$ for τ in (2.6) gives

$$0.40\ F_y = \frac{2\ P}{\pi\ d_p^2}$$

or

$$d_p = \sqrt{\frac{2\ P}{0.40\ \pi\ F_y}} = 1.26\sqrt{\frac{P}{F_y}} \tag{2.7}$$

The hole diameter for a pin is the pin diameter plus 1/16 in or 1.5 mm.

We shall assume that the available diameters for pins are the same as the available diameters for circular bars given in Table 1.4 (page 19). Our metric circular bars start with a 6 mm diameter and increase in 2 mm increments through a diameter of 60 mm. The increments are 5 mm for diameters from 65 mm to 80 mm. USCS circular bars start with a 1/4 inch diameter and increase in 1/16 inch increments through a diameter of one inch. The increments are 1/8 inch for diameters between 1 and 3 inches and 1/4 inch increments for diameters greater than 3 inches.

Example 2.2 ■■■■

The pin in a clevis type connection must transfer an axial tension force of 28.6 kips. Determine the minimum pin diameter when the pin is made from a steel with F_y = 36 ksi.

Solution:

The pin diameter is determined by direct substitution into (2.7)

$$d = 1.26\sqrt{\frac{P}{F_y}} = 1.26\sqrt{\frac{28.6\ \text{kips}}{36\ \text{ksi}}} = 1.12\,\text{in}$$

A 1.125 inch diameter pin is adequate. ■■■■

2.1.4 Bearing Stresses

Bearing stresses are produced when two bodies come into contact. Bearing stresses occur between a pin and the material in a clevis, in bolted and riveted joints, and beneath the footings of a building. Bearing stresses are normal stresses and are calculated using

$$\sigma_{br} = \frac{P}{A_{br}} \tag{2.8}$$

where σ_{br} is the bearing stress, P is the applied load, and A_{br} is the bearing area. Equation (2.8) gives an average value. Bearing stresses are never uniformly distributed over the contact area. The exact distribution is unknown for most applications. An average value is the best we can do.

Our primary interest in bearing stresses is the normal stress which occurs between pins, bolts, or rivets and the plates they contact. If the bearing area is not large enough, then localized yielding can occur behind the fasteners and the holes become irregular in shape. The contact area between a pin and its hole defies description. The bearing stress distribution probably has a maximum value along the line of loading and decreases to zero somewhere along the hole circumference. Since the exact distribution is very difficult to determine, engineers use a simple estimate for the contact area and define a safety factor based on data from experimental studies of pinned joints. The bearing stress behind a pin is calculated using

$$\sigma_{br} = \frac{P}{A_{br}} = \frac{P}{td_p} = f_{br} \tag{2.9}$$

The bearing area A_{br} is a projected area calculated using the product of the pin diameter d_p and the plate thickness t. The quantity f_{br}, using the lowercase f for the calculated value, denotes the calculated bearing stress.

The bearing stress equation (2.9) combined with an allowable stress value can be used to develop an equation for the plate thickness. The allowable bearing stress for steel when the connection consists of a single pin is

$$F_{br} = 0.90F_y \tag{2.10}$$

Note that the uppercase F denotes the allowable value.

The thickness of a plate with a single pin hole can be determined by rearranging (2.9) and (2.10) into

$$t = \frac{1.11P}{d_p F_y} \tag{2.11}$$

Since the value of t is a minimum, a plate with a larger thickness is used.

Example 2.3

The 1.125 inch diameter pin from Example 2.2 supports an axial loading of 28.6 kips. Determine the minimum thickness for the primary member in the clevis system when the member is made from a steel with F_y = 36 ksi.

Solution:
The required member thickness is given by (2.11). Substituting for the parameters gives

$$t = \frac{1.11P}{d_p F_y} = \frac{1.11(28.6\,\text{kips})}{(1.125\,\text{in})(36\,\text{ksi})} = 0.784\text{ in}$$

The member must be 0.8125 (13/16) inch thick since bar stock comes in 1/16 inch increments.

2.1.5 Plug Shear

Another type of failure in a bolted or riveted joint occurs when there is not enough material between the fastener and the end of the member. The material behind the fastener fails in shear. This failure is another situation where the shear stress distribution is complicated and unknown. The engineer uses an average value calculation and a safety factor based on experimental data to determine the distance between the fastener and the end of the member. The average shear stress is determined by assuming a plug of material is pushed out at the end of the member (Figure 2.5). The curvature of the hole is neglected and the shortest distance to the end of the member is used in the analysis.

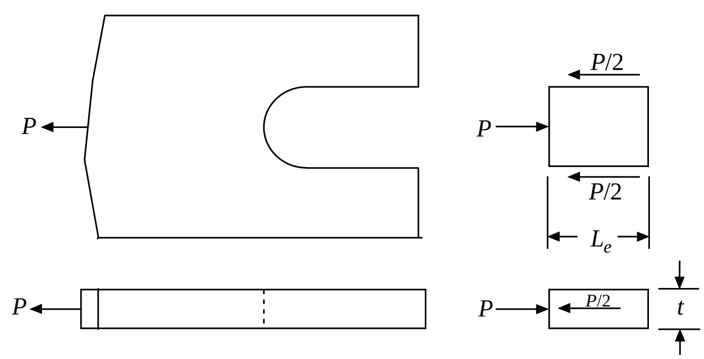


Figure 2.5 Plug shear failure at the end of a member.

The average shear stress on the material behind a pin is the force divided by the shear area

$$f_v = \frac{P}{A_v} \tag{2.12}$$

The contact force from the pin is resisted by shear stresses acting on two surfaces. The surfaces have a thickness t and a length L_e. The calculated shear stress is

$$f_v = \frac{P}{2tL_e} \tag{2.13}$$

The allowable shear stress for block shear behind a bolt or rivet is 0.25 F_u. An equation for the edge distance can be developed by replacing f_v in (2.12) with $0.25F_u$ to obtain

$$L_e = \frac{P}{2tf_v} = \frac{P}{2t\left(0.25F_u\right)} = \frac{2P}{tF_u} \tag{2.14}$$

This equation matches the minimum distance requirement given by AISC ASD (Equation J3-6, page 5-75).

The AISC ASD manual also recommends that the center-to-center distance of bolts shall not be less than three diameters, 3d, and the end distance in the line of the force shall not be less than 1.5d. These distance values are based on the clearance required to tighten the bolts. The AISC ASD manual (page 5-76) also gives a table for minimum edge distances as a function of plate thickness. The table is not reproduced here.

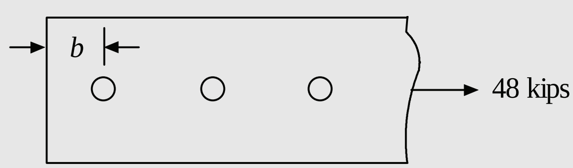
Equation (2.14) gives the edge distance behind the last fastener where the load is transferred by several bolts or rivets in a line. Pin-connected members such as plate links and eyebars, discussed in Chapter 6, have a single fastener. The allowable shear stress behind the fastener for these connections is $F_v = 0.20F_y$. Substitution of this value into (2.13) gives

$$L_e \geq \frac{P}{2tf_v} = \frac{P}{2t(0.20F_y)} = \frac{2.5P}{tF_u} \tag{2.15}$$

which is 20% greater than the minimum edge distance for the last fastener in a group of bolts or rivets.

Example 2.4

A rectangular bar 3.25 inches wide and 0.750 inches thick supports an axial tension load of 48 kips. The load is transferred to the connecting plates using three 0.750 inch diameter bolts placed in line. Determine the minimum distance from the centerline of the last hole to the end of the member denoted by b. The bar is made from A36 steel.



Solution:
Assuming that each bolt transfers an equal portion of the load, the axial force pushing against the plug of material at the end of the member is P = 48/3 = 16 kips. The material properties for the bar are F_y = 36 ksi and F_u = 58 ksi. Using (2.14), the minimum edge distance is

$$L_e = \frac{2P}{tF_u} = \frac{2(16\text{ kips})}{(0.750\text{ in})(58\text{ ksi})} = 0.736\text{ in}$$

Since the required dimension is from the centerline of the hole, we must add one-half the hole diameter which is 1/16 (0.0625) inch larger than the bolt diameter. The minimum distance b is

$$b = L_e + \frac{d_h}{2} = 0.736 + \frac{1}{2}\left(\frac{13}{16}\right) = 1.14\text{ in}$$

This value would probably be rounded up to the nearest 1/8 increment which produces a design value of 1.25 inch. This value must be compared with 1.5d which is

$$1.5d = 1.5\left(\frac{3}{4}\right) = 1.125 < 1.25 = L_e$$

The value of b = 1.25 inch satisfies all of the requirements. ■■■■

2.2 Axial Force Diagrams

The components of most structural frames are subjected to axial forces as well as shear forces and bending moments. Discussions of shear force and bending moment diagrams are included in basic mechanics courses. It is appropriate to discuss the construction of axial force diagrams, however, because these diagrams are not covered in most mechanics courses. A twisting moment diagram, which is very similar to the axial force diagram, is discussed in Chapter 9.

Consider the straight member subjected to three axial forces as shown in Figure 2.6. The bar is #182, a 64×64×3.0 light square tube with an area of 737 mm^2. When we have a member with these types of loadings, we usually want to know the magnitude of the maximum tension stress, the maximum compressive stress, and the relative change in length. Each of these calculations is relatively easy to do once we have an axial force diagram. An axial force diagram is constructed the same way we learned how to construct shear force and bending moment diagrams: The member is divided into segments and we determine the axial force required to keep the free body in equilibrium. The internal axial force is positive and produces a tension stress when the axial force required for equilibrium is directed away from the member. The internal axial force is negative and produces a compression stress when the axial force required for equilibrium is directed into the member. The procedure is illustrated using the free body diagrams shown in Figure 2.7. The first step is to replace the support by the axial force required to maintain equilibrium. In this case, a 20 kN force acting at the left end represents the support. A free body diagram of the member with the four axial forces is shown at the top of Figure 2.7.

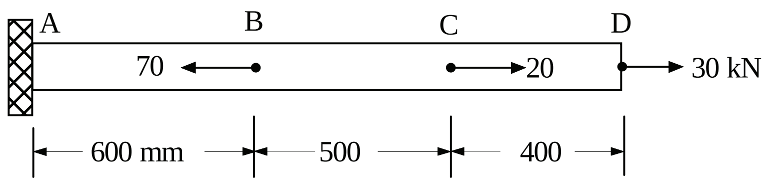


Figure 2.6 A straight member subjected to axial forces.

The first free body diagram has a cut between locations A and B. The segment has 20 kN axial forces acting inward on each face and this segment is in compression. The free body diagram looks the same regardless of the location of the cut between A and B, therefore, the 20 kN compression force is acting at every point between A and B.

The second free body has a cut between locations B and C. This diagram has applied loads of 20 kN to the right and 70 kN to the left, therefore, there must be a 50 kN axial force acting to on the right face to maintain equilibrium. The 50 kN force is acting away from the surface and produces a tension stress. Every location between B and C is subjected to a 50 kN tension force.

The third free body has a cut between locations C and D. The free body has two 20 kN forces acting to the right and the 70 kN force acting to the left. A 30 kN force acting away from the right face is required for equilibrium. Every location between C and D is subjected to a 30 kN force which produces tension stresses.

The final force diagram for this member is shown at the bottom of Figure 2.7. Tension is plotted as positive and compression is plotted as negative. The maximum compressive force is 20 kN and the maximum tension force is 50 kN. These force values are used to calculate the maximum normal stresses. The maximum tension stress is

$$\sigma_t = \frac{P}{A} = \frac{50\ \text{kN}}{737\ \text{mm}^2} = 67.8\ \text{MPa} \tag{2.16}$$

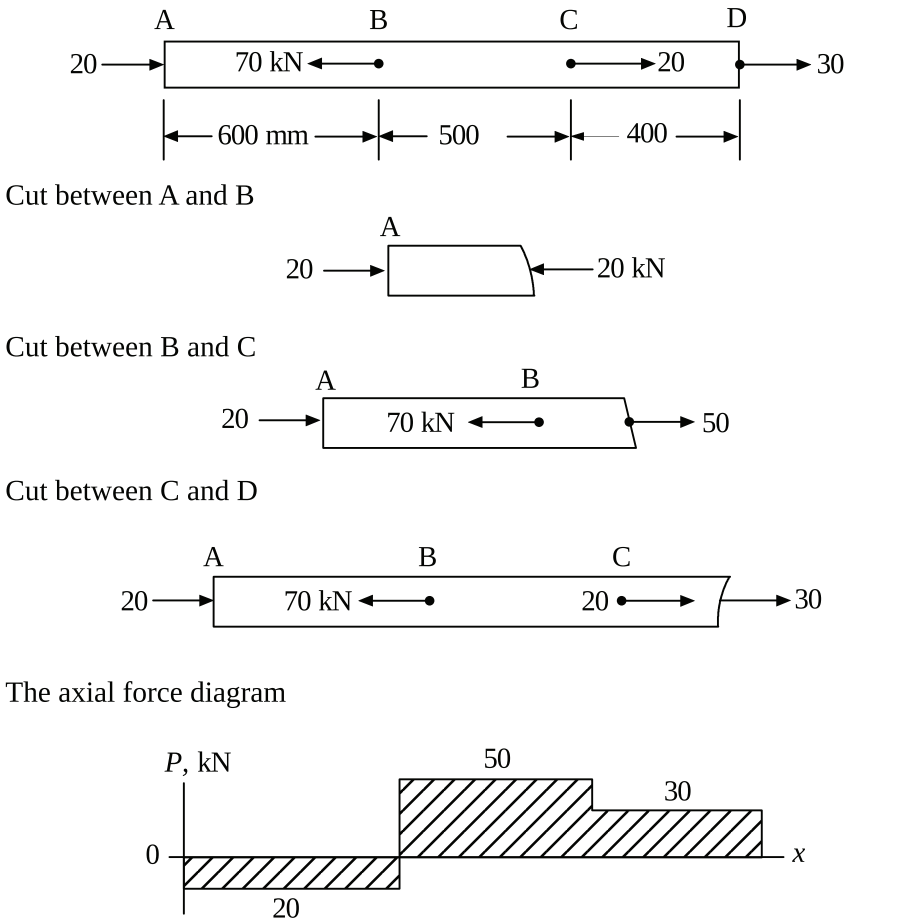


Figure 2.7 Free body diagrams for a bar with axial forces.

while the maximum compression stress is

$$\sigma_c = \frac{P}{A} = \frac{20 \text{ kN}}{737 \text{ mm}^2} = 27.1 \text{ MPa} \tag{2.17}$$

The relative change in length between A and D, Δ, is given by

$$\Delta = \sum \frac{P\,L}{A\,E} \tag{2.18}$$

where P is the axial force, L is the length over which the axial force is applied, A is the cross-sectional area, and E is the modulus of elasticity. The value for P can be obtained from the axial force diagram and has the same sign as the value on the axial force diagram. Substituting the information for the problem under discussion into (2.18) gives

$$\sum \frac{PL}{AE} = \frac{(-20 \text{ kN})(600 \text{ mm}) + (50\ )(500\ ) + (30\ )(400\ )}{(737 \text{ mm}^2)(200 \text{ kN/mm}^2)} = 0.170 \text{ mm}$$

An axial force member similar to the one in Figure 2.7 is displayed in Figure 2.8 with the axial force diagram shown directly below it. If we analyze several more straight members with a mixture of axial forces, we would observe similar results. The diagram jumps downward then the external force acts to the right and jumps upward when the external force acts to the left. The magnitude of each jump is equal to the magnitude of the applied force. The jumps in the axial force diagram are similar to those in a shear force diagram when a beam is subjected to concentrated forces. The rules for constructing an axial force diagram are shown schematically below the completed diagram.

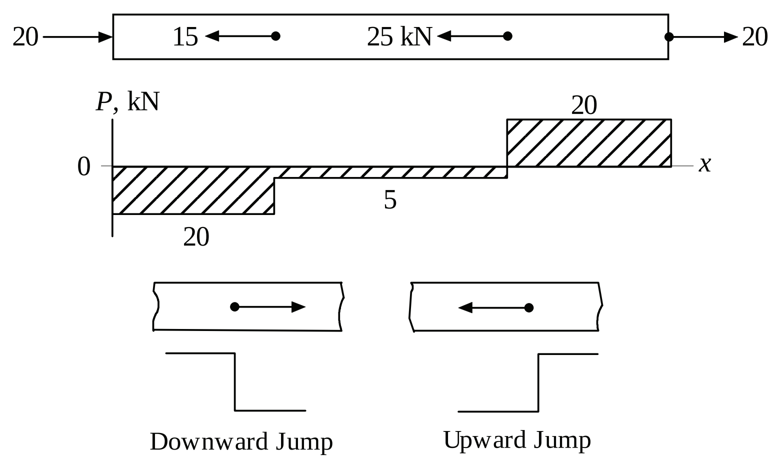


Figure 2.8 The direction of the jumps for an axial force.

2.3 Normal and Shear Stresses in Beams

A beam is a structural member that is designed to resist forces acting perpendicular to its longitudinal axis. The beam transfers the applied loads to remote supports by developing internal shear forces and bending moments. Most beams are planar structures because all of the loads act in a single plane and all deflections occur in the same plane. There are many different types of beams. Classification by the type of support gives simply supported beams, cantilever beams, beams with overhangs, and continuous beams.

Classification by the cross section gives beams with two axes of symmetry, a vertical axis of symmetry and shapes with no symmetry. Beams can also be straight or curved. This discussion is limited to straight beams with a vertical axis of symmetry. The resultant of each applied load must lie in the plane of symmetry, which eliminates twisting of the member about the *x* axis. The displacement method of structural analysis has no difficulty with fixed, pinned, or roller supports, therefore, there are no restrictions on how the beam is supported.

Transverse loads applied to a member in the *x*-*y* plane generate an internal shear force, $V(x)$, and an internal bending moment, $M_z(x)$. The shear force is parallel to the *y* coordinate direction while the bending moment acts about the *z* axis. The bending moment generates a normal stress component σ_{xx} that has its maximum values at the upper and lower surfaces and is zero at the neutral axis (NA) (Figure 2.9).

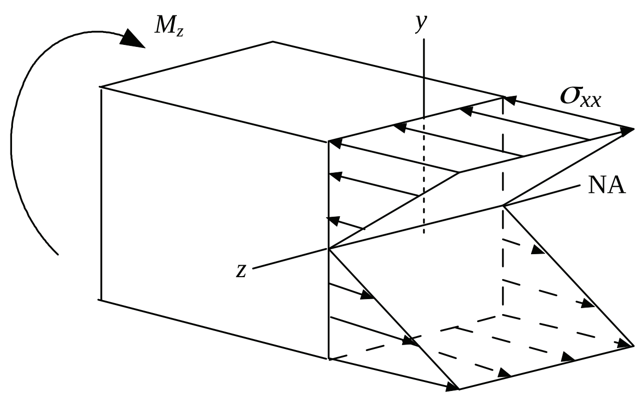


Figure 2.9 A beam segment showing the normal stress distribution.

The neutral axis for a straight beam coincides with the centroidal axis of the cross section. The normal stress equation is

$$\sigma_{xx} = -\frac{M_z y}{I_{11}} \tag{2.19}$$

where M_z is the magnitude of the internal bending moment about the *z* axis, *y* is the distance from the neutral axis to the point of interest, and I_{11} is the area moment of inertia about the 1-1 axis. A positive value for *y* is upward from the neutral axis. The variable I_{11} in (2.24) is often shortened to *I*. The use of *M* and *I* without subscripts implies a moment about the *z* axis and I_{11} in this book.

Equal maximum normal stress values occur at the upper and lower surfaces when a beam has two axes of symmetry. Defining the beam depth as h, then $c = h/2$ is the distance to each surface from the neutral axis and c can be substituted for y in (2.19) to obtain the equation for σ_{max} which is renamed f_b in the engineering design literature

$$\sigma_{max} = \frac{Mc}{I} = \frac{M}{\left(\frac{I}{c}\right)} = \frac{M}{S_b} = f_b \tag{2.20}$$

The minus sign in (2.19) is deleted for ductile materials because a material failure is not related to whether the normal stress is in tension or compression. The quantity I/c is the ratio of two section properties and is defined as S_b. This section property is used to calculate the maximum normal stress in a beam with the values for the respective axes denoted by S_{b1} and S_{b2}. The section property S_b is occasionally referred to as the section modulus for flexure stress. It is denoted by S without a subscript in the AISC ASD manual.

The internal shear force generates a shear stress distribution that is parabolic (Figure 2.10). The shear stress is zero at the top and bottom surfaces and has its maximum value at the neutral axis for sections composed of rectangles. A positive shear stress is upward on the left face and downward on the right face. The shear stress distribution is calculated using

$$\tau = \frac{VQ}{Ib} \tag{2.21}$$

where V is the magnitude of the shear force, $Q = A\overline{y}$ for the area above the point of interest, I is the area moment of inertia and b is the total thickness of the cross section at the point of interest. The quantity b is twice the wall thickness for tube sections.

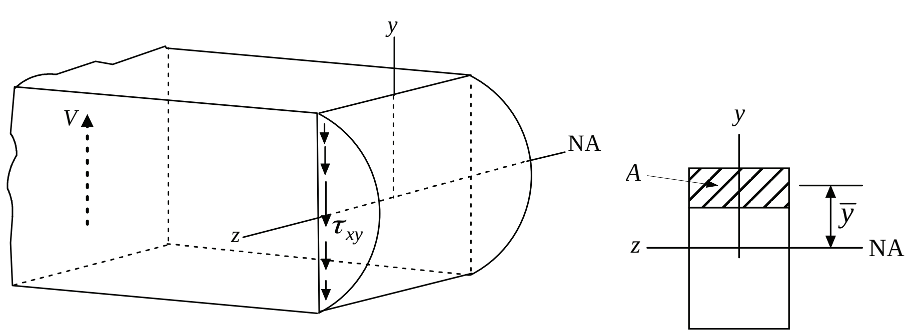


Figure 2.10 The shear stress distribution in a beam and the variables A and $\overline{y}$ in $Q = A\overline{y}$.

Since the maximum shear stress occurs at the neutral axis, the area in $Q = A\bar{y}$ is the area above the neutral axis and $\bar{y}$ is the distance from the neutral axis to the centroid of the area (Figure 2.10). Since each of *Q*, *I* and *b* are section properties, they can be grouped together and (2.21) can be written as

$$\tau_{max} = \frac{V}{\frac{Ib}{Q}} = \frac{V}{S_v} = f_v \tag{2.22}$$

where f_v is our notation for the maximum shear stress and S_v is the section property used to calculate its value. There is a value of S_v for each axis, S_{v1} and S_{v2} since there are two area moment values, I_{11} and I_{22}.

The maximum shear stress in a solid rectangular bar has the simple form of

$$\tau_{max} = \frac{3V}{2A} \tag{2.23}$$

where *A* is the cross-sectional area of the bar. Since the area is independent of the orientation, S_v is identical for each axis of a rectangular bar and is

$$S_v = \frac{2A}{3} \tag{2.24}$$

Six section properties are required to calculate both the normal and shear stresses, I_{11}, S_{b1}, and S_{v1} or I_{22}, S_{b2}, and S_{v2}. The area moments, I_{11} and I_{22}, are required for the calculation of S_b and S_v about each axis and are also required in beam deflection evaluations. Since the area moment is the primary section property for beams, a brief review of its evaluation is in order. The area moment of inertia is defined by the integral in (1.8). We are primarily concerned with the area moment about the horizontal *z* axis which is

$$I_{11} = \int_A y^2 dA \tag{2.25}$$

This integral arises during the derivation of the normal stress equation (2.19) and has been evaluated for standard shapes. The equations for some of these shapes are given in Figure 1.11 (p. 23). The evaluation of the section properties for the rectangular tube discussed in Section 1.2 is repeated here and expanded to include S_{b1} and S_{v1}.

Example 2.5

Evaluate the cross-sectional area, A, and the section properties I_{11}, S_{b1}, and S_{v1} for a rectangular tube with 5 by 8 inch outside dimensions, a 0.500 inch wall thickness, and square corners.

Solution:

Starting with the area,

$$A = 8\text{ in}(5\text{ in}) - 7\text{ in}(4\text{ in}) = 40 - 28 = 12\text{ in}^2$$

The area moment of inertia is

$$I_{11} = I_{solid} - I_{hole} = \left(\frac{bh^3}{12}\right)_{solid} - \left(\frac{bh^3}{12}\right)_{hole}$$

$$I_{11} = \frac{(5\text{ in})(8\text{ in})^3}{12} - \frac{(4\text{ in})(7\text{ in})^3}{12} = 99.0\text{ in}^4$$

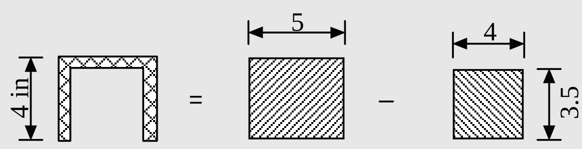
The section property for calculating the maximum normal stress in bending is the area moment of inertia divided by one-half of the beam depth

$$S_{b1} = \frac{I_{11}}{c} = \frac{99\text{ in}^4}{\left(\frac{8\text{ in}}{2}\right)} = 24.8\text{ in}^3$$

The section property S_{v1} for calculating $\tau_{max} = f_v$ is

$$S_{v1} = \frac{I_{11}b}{Q}$$

where $Q = A\bar{y}$ for the area above the neutral axis and b is the web thickness along the 1-1 axis. The value of Q is most easily evaluated using the composite area method. Visualize the region above the neutral axis as a 5 by 4 inch solid minus a 4 inch wide by 3.5 inch high hole.



The calculation of Q is

$$Q = A\bar{y}\big|_{solid} - A\bar{y}\big|_{hole}$$

$$Q = (5\text{ in})(4\text{ in})\left(\frac{4\text{ in}}{2}\right) - (4\text{ in})(3.5\text{ in})\left(\frac{3.5\text{ in}}{2}\right) = 15.5\text{ in}^3$$

and $$S_{v1} = \frac{I_{11}b}{Q} = \frac{(99.0\text{ in}^4)(2)(0.50\text{ in})}{15.5\text{ in}^3} = 6.39\text{ in}^2$$

■■■■

2.3.1 Section Properties for Composite Shapes

There are four composite sections that are often used for beams in machinery systems. The first is a structural tube with thin rectangular plates welded to the top and bottom surfaces (Figure 2.11a). The base section has an adequate S_{b1} value except in a region near the maximum bending moment. The combination of the plates and the base section provide the required S_{b1} value at the location of the maximum bending moment. The second composite section consists of vertical plates welded to the sides of a tube section (Figure 2.11b). The plates provide additional strength at the corners of rigid frames where large bending moments can exist and are used to provide additional thickness at pins where bearing stresses may govern the design. The third composite section is made using a pair of vertical plates and a pair of tube sections (Figure 2.11c). This combination provides an easy way to make tapered beams and is used in the lifting mechanism on some waste collection trucks. The upper-and-lower edges of the vertical plates are aligned with the centerline of circular sections but they could be located anywhere on the sides of square or rectangular tubes. Another way to make a tapered beam is to use channel sections as the flanges and tapered plates as the sides (Figure 2.11d). The side plates are usually flush with the outer surface of the channel sections.

Composite sections have components whose centroidal axis does not coincide with the centroidal axis of the entire section. The parallel-axis theorem must be used to calculate the area moment of inertia when this situation occurs. The parallel-axis theorem states that the area moment of an area about an axis that does not coincide with its centroidal axis is

$$I_{11} = I_{z'z'} + Ad^2 \tag{2.26}$$

where $I_{z'z'}$ is the area moment of inertia of the shape about its own centroidal axis, A is the area of the shape, and d is the distance between the centroidal axis of the shape ($z'z'$) and the axis about which the area moment is being calculated, in this case the 1-1 axis (Figure 2.12). The area moment calculations for a tube section with plates attached to the top and bottom surfaces are illustrated in Example 2.6.

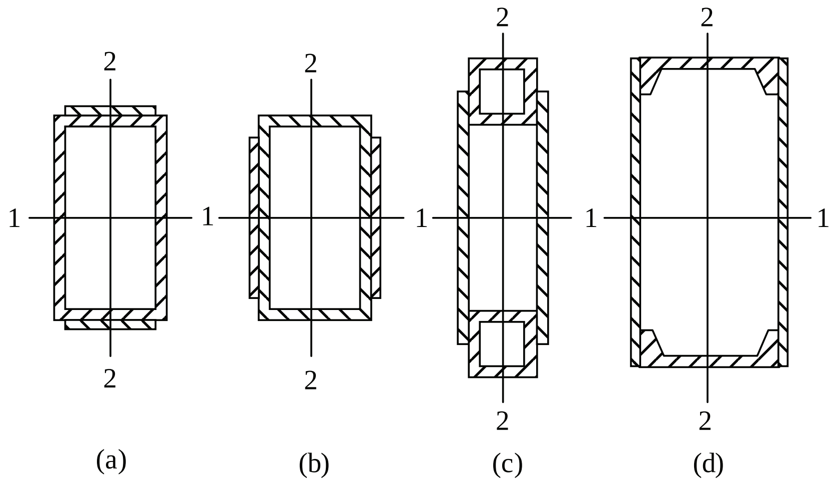


Figure 2.11 Four types of composite sections.

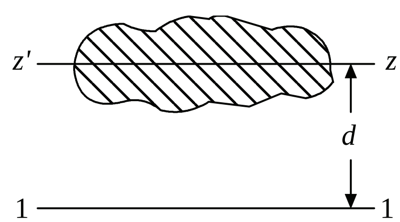
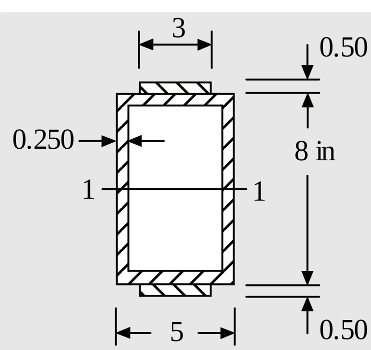


Figure 2.12 The distance *d* in the parallel-axis theorem.

Example 2.6

Rectangular plates three inches wide and one-half inch thick are welded to the top and bottom surfaces of #119, a 5×8×0.2500 RST. Calculate the section properties I_{11}, S_{b1}, and S_{v1} for the composite section about the 1-1 axis.



Solution:

The parallel-axis theorem must be used because the centroidal axis of each plate does not coincide with the centroidal axis of the tube. The area moment for the composite section is the area moment of the tube plus the area moment of the plates,

$$I_{11} = I_{tube} + I_{plates}$$

The area moment value for the tube about its centroidal axis is 52.6 in^4 (Appendix Table B2.2). The area moment of the two plates about the 1-1 axis is

$$I_{plates} = 2(I_{z'z'} + Ad^2) = 2\left(\frac{bh^3}{12} + Ad^2\right)$$

$$I_{plates} = 2\left(\frac{(3\,\text{in})(0.5\,\text{in})^3}{12} + (3\,\text{in})(0.50\,\text{in})(4.25\,\text{in})^2\right) = 54.25\,\text{in}^4$$

The area moment of inertia for the complete section is

$$I_{11} = I_{tube} + I_{plates} = 52.6 + 54.25 = 107\ \text{in}^4$$

The S_{b1} value is

$$S_{b1} = \frac{I_{11}}{c} = \frac{107\ \text{in}^4}{\left(\frac{9\ \text{in}}{2}\right)} = 23.8\,\text{in}^2$$

where c is half the depth of the complete section which is 9 in.

The section property for calculating f_v is

$$S_{v1} = \frac{I_{11}b}{Q}$$

where I = 107 in^4, the value evaluated above, while b = 2(0.250) = 0.500 in, and $Q = A\bar{y}$ for the area above the neutral axis. The quantity Q can be evaluated using the composite rule for calculating the centroid giving

$$Q = Q_{tube} + Q_{plate} = A\,\bar{y}\big|_{tube} + A\,\bar{y}\big|_{plate}$$

The quantity Q_{tube} can be evaluated by noting that S_{v1}= 3.27 in^2 for #119. Rearranging the equation for S_{v1} gives

$$Q_{tube} = \frac{I\,b}{S_{v1}} = \frac{(52.6\ \text{in}^4)(0.500\ \text{in})}{3.27\ \text{in}^2} = 8.04\ \text{in}^3$$

The quantity Q_{plate} is

$$Q_{plate} = A\bar{y} = (3\,\text{in})(0.500\,\text{in})(4.25\,\text{in}) = 6.375\,\text{in}$$

The section property for evaluating f_v is

$$S_{v1} = \frac{I\,b}{Q} = \frac{(107\ \text{in}^4)(0.500\ \text{in})}{(8.04 + 6.375)\ \text{in}^3} = 3.71\ \text{in}^2$$

■■■■

The S_{b1} value for the tube section alone is 13.2 in^3. The new S_{b1} value for the tube section with the plates is 23.8 in^3. The bending strength of the new section is 80% greater than the base section while the depth is increased only 12.5%. Increasing the bending strength by welding thin plates to the top and bottom surfaces of a base section is a common design practice.

An analysis of the values in the evaluation of I_{11} for the above example indicates that the area moment of the plates about their own centroidal axis is very small, 2(0.0625) in^4. This occurs because their height is less than one inch and the height value is cubed. It is common engineering practice to neglect the area moment of the plates about their centroidal axis when their thickness is less than one inch. Neglecting the area moment of the plates reduces the calculation to

$$I_{11} = I_{tube} + 2A_{plates}\, d^2 \tag{2.27}$$

The area moment value for the above problem using this simplification is

$$I_{11} = 52.6 \text{ in}^4 + 2[(3 \text{ in})(0.5 \text{ in})(4.25 \text{ in})^2] = 107 \text{ in}^4$$

which is identical to the value obtained in Example 2.7.

The composite section consisting of a box or tube section with vertical plates welded on each side is relatively easy to analyze. Assuming that the plates are centered about the 1-1 axis,

$$I_{11} = I_{tube} + I_{plates} \tag{2.28}$$

where

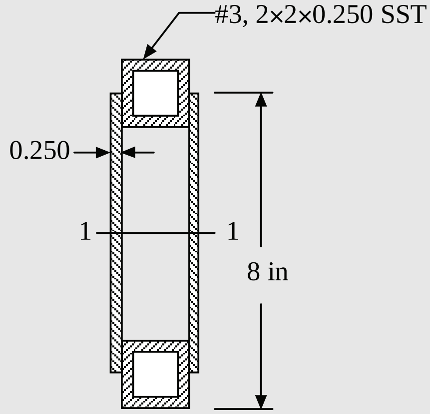
$$I_{plates} = 2\left(\frac{bh^3}{12}\right) \tag{2.29}$$

The calculation of I_{plates} is a special case of the parallel-axis theorem where d is zero and $I_{11} = I_{z'z'}$.

The parallel-axis theorem is also used to calculate the area moment for composite sections made using vertical plates and tube sections. The area moment of the tube sections about their centroidal axis $I_{z'z'}$ is not negligible and must be included in the calculations. The evaluation of the section properties for a shape made using vertical plates and tubes is illustrated in the following example.

Example 2.7

A pair of rectangular plates, 0.250×8.00 inches and oriented vertically, are used with a pair of square structural tubes, #3, 2×2×0.2500, to form a composite section. The plates extend to the horizontal centerline of the tubes. Calculate the properties I_{11}, S_{b1}, and S_{v1} for the final section.



Solution:

The area moment for the composite section is the area moment for the pair of plates plus the area moment for each tube section about the 1-1 axis.

$$I_{11} = I_{tube} + I_{plates}$$

Considering the plates first,

$$I_{plates} = 2\left(\frac{bh^3}{12}\right) = 2\left[\frac{(0.250\ \text{in})(8\ \text{in})^3}{12}\right] = 21.3\ \text{in}^4$$

Using the parallel-axis theorem for the tubes,

$$I_{tube} = 2(I_{z'z'} + Ad^2) = 2[0.77\ \text{in}^4 + (1.59\ \text{in}^2)(4\ \text{in})^2] = 52.4\ \text{in}^4$$

where $I_{z'z'} = 0.77\ \text{in}^4$ and $A = 1.59\ \text{in}^2$ are obtained from Appendix Table B2.1. The area moment value is

$$I_{11} = 21.3\ \text{in}^4 + 52.4\ \text{in}^4 = 73.7\ \text{in}^4$$

The section property for calculating the normal stress, S_{b1}, is

$$S_{b1} = \frac{I_{11}}{c} = \frac{73.7\ \text{in}^4}{\left(\frac{10\ \text{in}}{2}\right)} = 14.7\ \text{in}^3$$

The final section has an overall height of 10 in since one-half of each tube extends beyond the plates and each tube is 2 in on a side.

The section property for calculating f_v is

$$S_{v1} = \frac{I\,b}{Q}$$

where I = 73.7 in^4, the value evaluated above, and b is the total thickness of the two plates or 0.500 in. The quantity Q is

$$Q = Q_{tube} + Q_{plates}$$

where

$$Q_{plates} = A\bar{y} = 2(0.250 \text{ in})(4.00 \text{ in})(2.00 \text{ in}) = 4.00 \text{ in}^3$$

while

$$Q_{tube} = A\bar{y} = (1.59 \text{ in}^2)(4.00 \text{ in}) = 6.36 \text{ in}^3$$

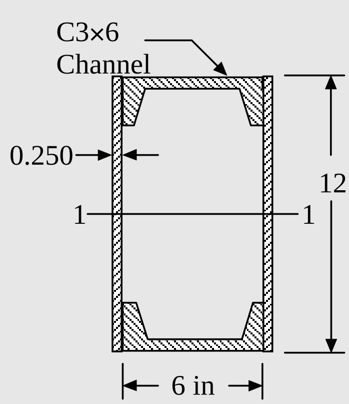
The area in $Q = A\bar{y}$ is the cross-sectional area above the neutral axis, therefore only one tube is included in the calculations. The section property for evaluating f_v is

$$S_{v1} = \frac{I\,b}{Q} = \frac{(73.7 \text{ in}^4)(0.500 \text{ in})}{(4.00 + 6.36) \text{ in}^3} = 3.56 \text{ in}^2$$

The properties for a composite section made using vertical plates and channels also require the use of the parallel-axis theorem. The properties of 10 channel sections that could be used as flanges are given in Appendix Table B4.1. The metric sections are direct conversions of the USCS sections (AISC, 1992).

Example 2.8

A pair of C3×6 channels are used as flanges for a composite box section. The vertical plates are 12 in high and 0.250 in thick. The upper and lower edges of the plates are flush with the flat surface of each channel. Evaluate I_{11}, S_{b1}, and S_{v1} for the composite section.



Solution:

The area moment for the composite section is

$$I_{11} = 2\left(I_{plates} + I_{channels}\right) = 2\left(\frac{bh^3}{12}\right) + 2\left(I_{z'z'} + Ad^2\right)$$

The section properties for the C3×6 channel, Appendix Table B4.1, are A = 1.76 in^2, I_{11} = 0.305 in^4 = $I_{z'z'}$, and $\bar{y}$ = 0.455 inch below the flat side. The area moment for a single channel is

$$I_{ch} = I_{z'z'} + Ad^2 = 0.305\ \text{in}^4 + (1.76\ \text{in})\ [(6 - 0.455)\text{in}]2 = 54.4\ \text{in}^4$$

The final value of I_{11} is

$$I_{11} = 2\left(\frac{(0.250\ \text{in})(12\ \text{in})^3}{12} + 54.4\right) = 181\ \text{in}^4$$

while
$$S_{b1} = \frac{I_{11}}{c} = \frac{181\ \text{in}^4}{\left(\frac{12\ \text{in}}{2}\right)} = 30.1\ \text{in}^3$$

The section property for calculating f_v is

$$S_{v1} = \frac{I\,b}{Q}$$

where I = 54.4 in^4 (the value evaluated above) and b = 0.500 inch. The quantity Q is

$$Q = Q_{channel} + Q_{plates}$$

where

$$Q_{plates} = A\bar{y} = 2(6.00\ \text{in})(0.250\ \text{in})(3.00\ \text{in}) = 9.0\ \text{in}^3$$

while

$$Q_{channel} = A\bar{y} = (1.76\ \text{in}^2)(6 - 0.455)\ \text{in}) = 9.76\ \text{in}^3$$

The section property for evaluating the maximum shear stress is

$$S_{v1} = \frac{I\,b}{Q} = \frac{(181\ \text{in}^4)(0.500\ \text{in})}{(9.76 + 9.0)\ \text{in}^3} = 4.82\ \text{in}^2$$

2.4 Shear Force and Bending Moment Diagrams

Beam design requires knowledge of the magnitude of the internal shear force and bending moment values. This information is obtained by constructing shear force and bending moment diagrams. These diagrams are generally introduced in a statics course and discussed again in a mechanics of deformable solids course, so a detailed discussion of the topic is not provided here. Three examples are presented to illustrate the sign convention and to provide a brief review on how to construct the diagrams.

A free body diagram of a beam segment is presented in Figure 2.13 with the positive definitions for the distributed load, internal shear forces, and internal moments. A positive distributed load is directed in the positive y direction, and positive shear forces are upward on the left face and downward on the right face. Positive internal bending moments cause the beam to deflect such that it will hold water. The internal shear force and bending moment on the left face are denoted by V and M, respectively. The shear force and bending moment on the right face are approximated using the first two terms of a Taylor series. A summation of forces in the y direction yields

$$+\uparrow \sum F_y = V + (w\,dx) - \left[V + \left(\frac{dV}{dx} \right) dx \right] = 0$$

Simplification gives

$$\frac{dV}{dx} = w \tag{2.30}$$

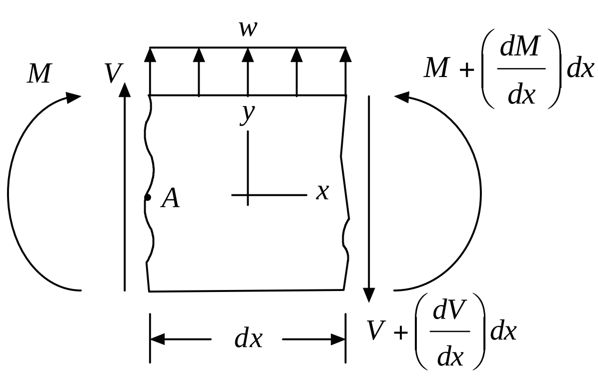


Figure 2.13 Free body diagram of a beam segment.

Summation of moments about A gives

$$\overset{+}{\circlearrowleft}\sum M_A = -M + (w\,dx)\left(\frac{dx}{2}\right) + M + \left(\frac{dM}{dx}\right)dx - \left[V + \left(\frac{dV}{dx}\right)dx\right]dx = 0$$

Rearranging this equation and discarding the second order terms $(dx)^2$ and $(dV)(dx)$ because they go to zero as dx goes to zero leaves

$$\frac{dM}{dx} = V \tag{2.31}$$

Equation (2.30) relates the slope of the shear force diagram to the distributed loading at a specific location while (2.31) relates the slope of the moment diagram to the value of the shear at a specific location. These equations are seldom evaluated numerically. The word form of each equation is used to assist with the construction of the individual diagrams. The word forms are listed here as a set of rules. The word "point" in these rules refers to a specific location on the beam.

Rule 1: The slope of the shear force diagram at a point is equal to the value of the distributed load at the same location.

Rule 2: The slope of the bending moment diagram at a point is equal to the value of the shear force at the same location.

Equations (2.30) and (2.31) can be integrated to obtain relationships that give the value of the shear force and bending moment at new locations. Integration between x_i and x_j yields

$$V_j = V_i + \int_{x_i}^{x_j} w(x)\,dx \tag{2.32}$$

and

$$M_j = M_i + \int_{x_i}^{x_j} V(x)\,dx \tag{2.33}$$

These two equations can be evaluated to obtain V_i and M_i but each integral is usually interpreted as the area under the appropriate curve between x_i and x_j. These equations also have a word form.

Rule 3: The change in the shear force between any two points is equal to the area under the distributed load diagram between the same two points.

Rule 4: The change in the bending moment between any two points is equal to the area under the shear force diagram between the same two points.

Two other rules complete the quick review of internal shear force and bending moment diagrams.

Rule 5: The shear force diagram has a step change at the locations of a concentrated force. The diagram jumps in the direction of the concentrated force an amount equal to the magnitude of the force.

Rule 6: The bending moment diagram has a step change at the location of a concentrated moment. The diagram jumps an amount equal to the magnitude of a concentrated moment. The jump is upward for a clockwise moment and downward for a counterclockwise moment.

This quick review is concluded by discussing the shear force and bending moment diagrams for three example problems. Several beam loadings are given at the end of this chapter and the reader should construct shear force and bending moment diagrams until they are satisfied with their ability to do so. Accurate construction of the shear force and bending moment diagrams is an essential part of beam design.

The first example, Figure 2.14, is a simply supported beam with two concentrated forces. The left end has a pin support while the other support is a roller. The reaction forces were obtained using the equations of statics. The horizontal force at the pin is zero because there are no horizontal forces applied to the beam.

The shear force and bending moment diagrams are constructed under a free body diagram of the beam that gives the support forces. The shear force diagram is drawn first because it is related to the beam loading. The bending moment diagram is constructed under the shear force diagram because its slope and numerical values are related to the shear force values. All of the forces acting on the beam in Figure 2.14 are concentrated forces. This includes the support forces. The shear force diagram jumps in the direction of each force an amount equal to the magnitude of the force. The shear force diagram consists of horizontal lines between the jumps because the distributed load is zero, $dV/dx = w = 0$, which implies a horizontal line. The dV/dx criterion comes from (2.30).

The bending moment diagram starts at zero and ends at zero because pin or roller supports located at the ends of a beam are not capable of developing a moment. The bending moment diagram consists of straight line segments because $dM/dx = V$ (2.31). The slope of the moment diagram at a specific location equals the value of the shear force at the same location. Two items should be considered when applying (2.31). First, is the shear force positive or negative? Second, is the shear force constant or changing? The shear force between locations A and B of Figure 2.14 is positive and constant. This means the bending moment diagram is a straight line with a positive slope. The value of the bending moment at B, 7.68 kip·ft, is the area under the shear force diagram between A and B. The word version of (2.30), Rule 4, can be placed in an equation form that uses the area under the shear force diagram, as

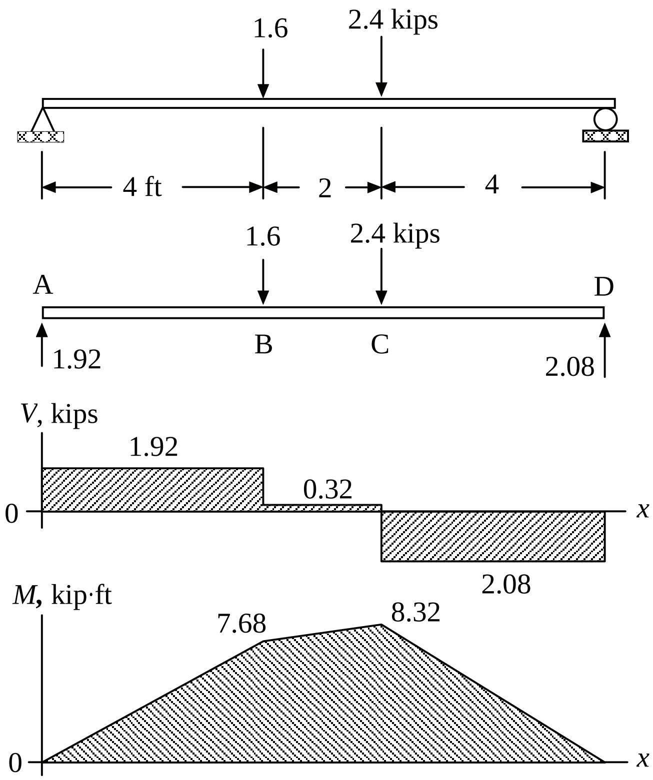


Figure 2.14 Shear force and bending moment diagrams for a beam subjected to concentrated forces.

$$M_B = M_A + A_{V_{A\to B}} = 0 + (1.92 \text{ kips})(4 \text{ ft}) = 7.68 \text{ kip·ft}$$

where $A_{V_{A\to B}}$ represents the area under the shear diagram between locations A and B. The moment value at the pin, M_A, is zero.

The bending moment diagram is a straight line with a positive slope between locations B and C because the shear force in this region is still positive and constant. The slope in region BC is less than the slope in region AB because the magnitude of the shear force is less in region BC than it is in region AB. The change in magnitude of the bending moment values between locations B and C is the area under the shear force between B and C. In equation form

$$M_C = M_B + A_{V_{B\to C}} = 7.68 \text{ kip·ft} + (0.32 \text{ kips})(2 \text{ ft}) = 8.32 \text{ kip·ft}$$

where M_B is the value calculated above.

The bending moment diagram between C and D is a straight line because the shear force in this region is also constant. The line has a negative slope, however, because the shear force is negative. The change in the moment value is

$$M_D = M_C + A_{V_{C\to D}} = 8.32 \text{ kip·ft} + (-2.08 \text{ kips})(4 \text{ ft}) = 0$$

The zero value at D is expected because of the roller support. The bending moment is zero at a pin support or a roller support only if they are located at the ends of the beam. Nonzero internal bending moments occur at these supports if a portion of a loaded beam extends beyond the support.

The second example, Figure 2.15, has a uniform distributed load acting over 80% of the length. The equations of statics give the vertical forces of 9.6 and 6.4 kips at locations A and C, respectively. The distributed load acts downward, which is negative according to the sign convention in Figure 2.13. Using (2.30), dV/dx = -2 kips/ft, because the slope is negative. The shear force diagram decreases at a rate of 2 kips/ft under the distributed load. The shear force diagram first jumps to 9.6 kips and then starts downward as a straight line decreasing 16 kips over the 8 ft length of the distributed load accounting for the -6.4 kips value at B. The shear force diagram is

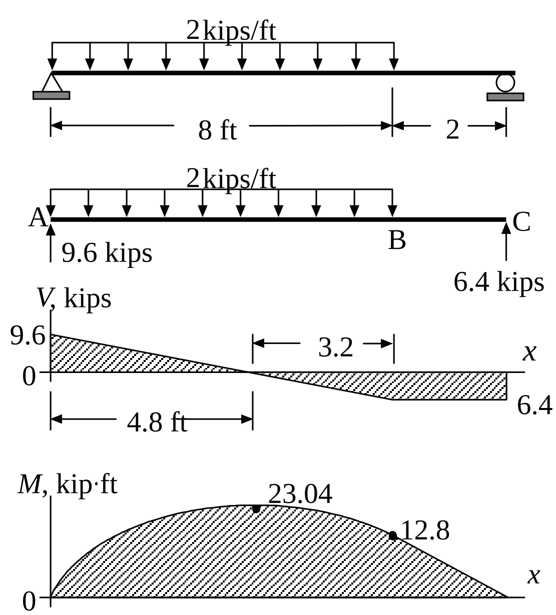


Figure 2.15 Shear force and bending moment diagrams for a beam with a distributed loading.

horizontal from B to C because the distributed load is zero in this region. The diagram jumps upward to 6.4 kips, returning to zero, at C.

The bending moment diagram can not be constructed until the zero point of the shear force diagram is known. The value of 4.8 ft is obtained by dividing the shear value of 9.6 kips by the rate of change, 2 kips/ft. The calculation is

$$L = \frac{V}{w} = \frac{9.6\text{ kips}}{2\text{ kip/ft}} = 4.8\text{ ft}$$

Draw a vertical line passing through the zero location of the shear force diagram before constructing the bending moment diagram.

The slope of the bending moment diagram at a specific location is the value of the shear force at the same location. The value of the shear force between A and B is continually changing, therefore, the slope of the moment diagram is continually changing. The moment value is a curved line. The slope of the moment diagram is zero where the shear force is zero and the slope is negative at all points to the right of the 4.8 foot value. The bending moment diagram is a straight line between B and C because the shear force is constant in this region.

Relative maximum and minimums bending moment values occur where $dM/dx = 0$. This criterion is satisfied at locations where the shear force diagram goes through zero. The maximum bending moment for this beam occurs 4.8 feet from the left support and is

$$M_D = M_A + A_{V_{A\rightarrow D}} = 0 + (9.6\text{ kips})(4.8\text{ ft})/2 = 23.04\text{ kip}\cdot\text{ft}$$

The bending moment at B is

$$M_B = M_D + A_{V_{D\rightarrow B}} + 23.04\text{ kip}\cdot\text{ft} + (-6.4\text{ kips})(3.2\text{ ft})/2 = 12.8\text{ kip}\cdot\text{ft}$$

The bending moment goes to zero at C because the negative area under the shear force diagram between B and C equals the 12.8 kip·ft value at B.

The third example, Figure 2.16, illustrates the jump in the bending moment diagram resulting from a concentrated moment. Concentrated moments are relatively common in machinery structures because the brackets for hydraulic cylinders are often attached to the top or bottom of a member rather than at the neutral axis. The vertical reaction forces are 0.6 kip at the left end and 1.4 kips at the right end. The shear force diagram is very similar to the one in Figure 2.14 because there is no distributed load, $w = 0$, and the shear force is a horizontal line between jumps. The bending moment diagram is a positive straight line between A and B because the shear force is positive and constant in this region. The moment value is

$$M_B = M_A + A_{V_{A\rightarrow B}} = 0 + (0.6\text{ kips})(4\text{ ft}) = 2.4\text{ kip}\cdot\text{ft}$$

Using Rule 6, the clockwise concentrated moment at B produces an upward jump in the moment diagram equal in magnitude to the value of the concentrated moment. The moment value just to the right of B is (2.4 kip·ft + 6 kip·ft) = 8.4 kip·ft. The 8.4 kip·ft value is easily checked because the bending moment diagram must decrease to zero at C. The area under the shear force diagram between B and C is -8.4 kip·ft and the moment at C is zero.

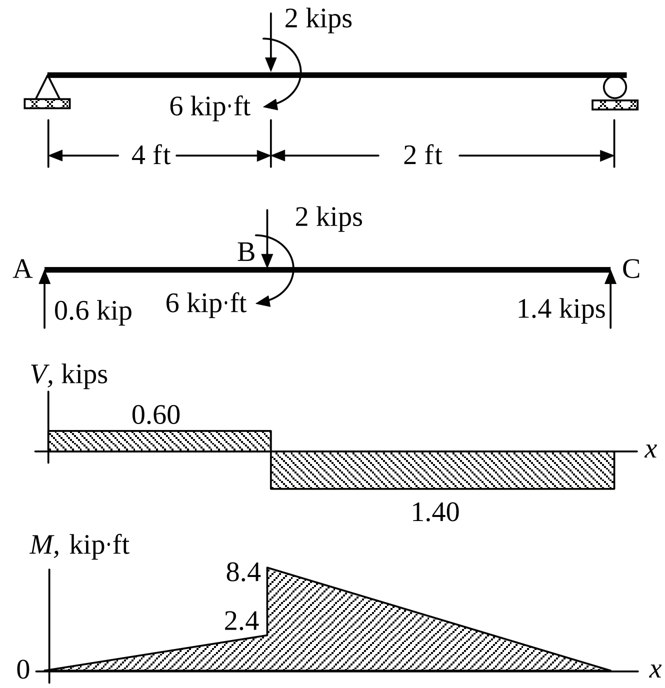


Figure 2.16 Shear force and bending moment diagrams for a beam with a concentrated force and a concentrated moment.

2.5 Combined Loading Diagrams

The axial force diagram was introduced in Section 2.2 and we have just finished reviewing shear force and bending moment diagrams. Many of the structural components in machines are subjected to axial forces as well as shear forces and bending moments. It is necessary to construct all three of these diagrams to determine which combined loading situations govern the design of a particular member. The construction of the combined loading diagrams differs very little from the previous discussions. The member is represented by a horizontal line. All of the forces and moments acting on it

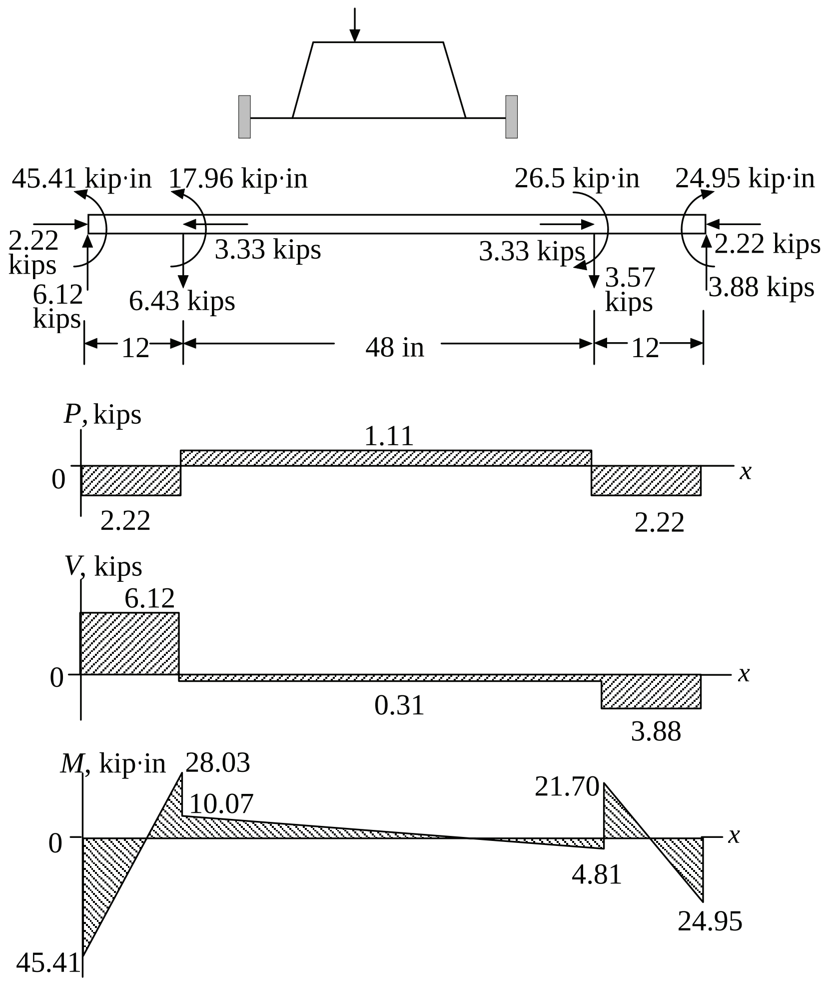


Figure 2.17 The combined loading diagrams for a structural component.

are placed in the appropriate locations and the three diagrams are constructed below the member. The axial force diagram is displayed first followed by the shear force and bending moment diagrams. An example is presented in Figure 2.17. The diagrams are for the bottom member of a rigid frame in a trailer irrigation system, which is shown schematically at the top of the figure. The concentrated moments occur because all of the joints are welded. Note that a portion of the member is subjected to the combined loading of an axial tension force and bending moments while other portions are subjected to the combined loading of an axial compression force and bending moments.

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Appendix A

Specifications and Code

This set of specifications covers the design of structural components made from circular, square, and rectangular tubes as well as circular, square, and rectangular bar stock as they occur in machines. These specifications have been compiled for educational purposes and contain accepted design procedures as given in the civil and mechanical engineering literature. These specifications do not include design criteria for channels, angles, and I-sections and do not apply to structural components that experience a large number of stress reversals during their lifetime. The information on member dimensions and materials is provided to simulate the design process and is not all inclusive.

Appendix A1 Design Basis

All structural members, connections, and connectors shall be proportioned so the stresses resulting from the working loads do not exceed the allowable stresses specified herein. The stresses in members, connections, and connectors shall be determined using accepted analysis procedures selected by the responsible engineer.

Appendix A2 Units

These specifications can be used with either the United States Customary System of units (USCS) or metric units (SI). The unit of length in calculations is the inch (USCS) and the millimeter (SI). The units for normal, shear, and yield stresses are kips per square inch (ksi) and megapascals (MPa). One kip is 1000 pounds. One megapascal is one Newton (N) per square millimeter. The material properties for steel are given in ksi or GPa. All calculations are done in inches or millimeters. The units for F_y in all equations in this design code are ksi or MPa.

Appendix A3 Materials and Dimensions

All structural components are made using steel whose elastic properties are:

USCS units

Elastic modulus $E = 29\,000$ ksi
Shear modulus $G = 11\,600$ ksi

SI units

Elastic modulus $E = 200$ GPa
Shear modulus $G = 80$ GPa

The structural components included in these specifications are:

Square structural tubes (SST)
Rectangular structural tubes (RST)
Light square tubes (LST)
Light rectangular tubes (LRT)
Circular tubes (CT)
Circular bars (CB)
Square bars (SB)
Rectangular bars (RB)
American standard channels (C)
Plate steel (PS)
A325 and A490 high strength bolts

The relationship between the structural shape and the type of steel is given in Appendix Table A3.1 The yield stress F_y and the minimum ultimate tensile stress F_u for each steel is given in Appendix Table A3.2. The acceptable size increments for circular, square, and rectangular bars are given in Appendix Table A3.3. The dimensions of shapes cut from plate steel changes in 1/8 inch or 2 millimeter increments. Even values are used in the metric system. Plate steel has the same thickness increments as rectangular bars. Threaded tension members have the same size increments and steel types as circular bars.

Appendix Table A3.1
The relationship between the type of steel and the structural shape.

Structural Shape	Type of Steel	Comments
Circular tubes	UNS G10180	Cold drawn seamless, low carbon
	ASTM A501	Hot formed tubes
Light tubes	AISI 1010	Made from No. 2 finished cold rolled strips
Structural tubes	ASTM A501	Hot formed tubes
	ASTM A500, grade B	Cold formed tubes
Circular bar stock	ASTM A36	Carbon structural steel
	UNS G10180	Case hardened cold drawn steel
Square bar stock	AISI M1020	General purpose merchant quality steel
	UNS G10180	Case hardened, cold drawn squares
Rectangular bar stock	ASTM A36	Carbon structural steel
	UNS G10180	Case hardened, cold drawn flats
Plate sheets	ASTM A36	Carbon structural steel
	ASTM A572	High strength-low alloy steel
Bolts	ASTM A325	High strength steel
	ASTM A490	High strength steel

Appendix Table A3.2
The yield stress and ultimate tensile stress for some steels and welding rods.

Type of Steel	Yield Stress, F_y	Ultimate Tensile Stress, F_u
AISI 1010	35 ksi 241 MPa	45 ksi 310 MPa
AISI M1020	35 ksi 241 MPa	63 ksi 434 MPa
ASTM A36	36 ksi 248 MPa	58 ksi 400 MPa
ASTM A501	36 ksi 248 MPa	58 ksi 400 MPa
ASTM A572	42 ksi 289 MPa	60 ksi 413 MPa
ASTM A500, grade B	46 ksi 317 MPa	58 ksi 400 MPa
UNS G10180	54 ksi 372 MPa	64 ksi 441 MPa
AISI 1040	55 ksi 379 MPa	87 ksi 600 MPa
E60 welding rod	--	60 ksi 414 MPa
E70 welding rod	--	70 ksi 483 MPa

Appendix Table A3.3
Dimensions for round, square, and rectangular bars.

Type	Units	Bar Size	Increment
Circular bars	USCS	1/4 through 1 inch 1 1/8 through 3 inches 3 1/4 through 4 inches	1/16 inch 1/8 inch 1/4 inch
	SI	6 through 27 mm 30 through 75 mm 81 through 105 mm	1.5 mm 3 mm 6 mm
Square bars	USCS	1/4 through 1 inch 1 1/8 through 3 inch 3 1/4 through 4 inch	1/16 inch 1/8 inch 1/4 inch
	SI	6 through 27 mm 30 through 75 mm 81 through 105 mm	1.5 mm 3 mm 6 mm
Rectangular bars	USCS	1/8 through 1/2 inch 5/8 through 1 1/8 inch 1 1/4 through 3 inch Widths:	1/16 inch 1/8 inch 1/4 inch 1/8 inch
	SI	6 through 27 mm 30 through 75 mm 81 through 105 mm Widths: Even values	1.5 mm 3 mm 6 mm 2 mm
Bolt diameter	USCS	1/2, 5/8, 3/4, 7/8, 1, 1 1/8 inch	
	SI	16, 20, 24, 30, 36 mm	

Appendix A4 Design for Axial Tension

This section applies to members subjected to axial tension caused by forces acting through the centroid of the cross section. The tension members can be attached using welds, threads, bolts, or pins. See Section A8 for the design of members subjected to the combined loading of axial tension and bending.

Appendix A4.1 Welded Members

The allowable normal stress F_t in welded members shall be the $0.60F_y$ on the gross section and $0.50F_u$ on the net section.

Appendix A4.2 Threaded Circular Bars

The allowable normal stress F_t in the net area of the thread shall be $0.45F_u$. The allowable normal stress F_t at a section removed from the threads shall be the minimum of $0.60F_y$ or $0.50F_u$.

The net area in the thread is calculated using the mean thread diameter which is given by

$$d_m = d - \frac{0.974}{N}$$

for Unified National Coarse and Fine threads where N is the number of threads per inch. The mean diameter for Coarse and Fine Pitch metric threads is

$$d_m = d - 0.938p$$

where p is the pitch.

Appendix A4.3 Threaded Bars with Upset Ends

A circular bar with upset ends shall be designed such that the net area in the threaded region exceeds the net area of the base rod. The allowable normal stress F_t in the net area of the thread shall be $0.45F_u$. The allowable normal stress F_t at a section far removed from the threads shall be the minimum of $0.60F_y$ or $0.50F_u$.

Appendix A4.4 Bolted Members

The allowable normal stress F_t shall not exceed $0.60F_y$ on the gross area nor $0.50F_u$ on the net area of a bolted tension member. The net area shall be determined by multiplying the net width w_n by the member thickness. All bolt holes shall be drilled. The bolt hole diameter shall be taken as 1/16 inch (1.5 mm) greater than the bolt diameter. Each bolt in a connection with multiple bolts carries a percentage of the axial force in proportion to its cross-sectional area divided by the total cross-sectional area of all the bolts. The critical net section is the vertical, diagonal, or zigzag line that gives the least net width. In no case shall the net width through a hole or a series of holes be greater than 85% of the gross width.

The net width for a series of holes extending across a member in any diagonal or zigzag line shall be computed by deducting from the gross width the sum of the diameters of all the holes in the line and adding the quantity $p^2/4g$ for each gage spacing in the chain. The distance p is the longitudinal spacing (pitch) of any two consecutive holes and g is the transverse spacing (gage) of the same two holes.

The bearing stress f_{br} between each bolt and the tension member shall be less than the allowable bearing stress F_{br} of $0.90F_y$. The bearing area is calculated using the bolt diameter times the member thickness. The minimum distance between the end of a member and a bolt shall be determined assuming failure by block shear behind the bolt. The allowable shear stress F_v for block shear is $0.25F_u$.

Appendix A4.5 Pin-Connected Members

The allowable normal stress F_t on the net section at pin holes in pin-connected plate links and eyebars is $0.45F_y$. The allowable bearing stress F_{br} on the contact region behind the pin is $0.90F_y$.

Pins: Pin-connected plates and eyebars shall be connected using a single pin in double shear. The allowable shear stress F_y for the pin is 0.40 F_y. The pin hole d_h shall not be more than the pin diameter plus 1/32 inch (1 mm). Pin diameters have the same size increments and steel types as circular bars.

Plate Links: Pin-connected plates shall have a uniform thickness and a uniform width. Other design requirements include:

1. The net area perpendicular to the applied load P on a section through the pin is $2at$ where a is the minimum edge distance from the pin hole to the outer surface on a line perpendicular to the applied load and t is the plate thickness.
2. The minimum edge distance between the bolt hole and a surface parallel to the direction of the member, a, shall be less than four times the thickness.
3. The pin hole diameter d_h shall be greater than 5/4 times the minimum edge distance.
4. The pin hole diameter d_h must be less than five times the plate thickness.
5. The calculated bearing stress behind the pin shall be less than the allowable bearing stress. The bearing area is the plate thickness times the pin diameter.
6. The distance parallel to the member from the back edge of the pin hole to the end of the member shall be greater than 4/3 times the minimum edge distance.

7. The corners beyond the pin may be cut at 45°. The minimum distance from the edge of the pin hole to the cut shall be greater than 4/3 times the minimum edge distance.

Eyebars: Eyebars shall be of uniform thickness, without reinforcement at the pin holes, and have circular heads whose periphery is concentric with the pin hole. The radius of the transition between the circular head and the eyebar body shall not be less than the diameter of the head. Other design requirements include:

1. The ratio of the net area through the pin A_2 and the main body A_1 shall satisfy $1.33 \leq A_2/A_1 \leq 1.50$.
2. The pin diameter d_p shall not be less than 7/8 of the width of the body.
3. The width-to-thickness ratio must be greater than two and less than eight.
4. The bearing stress between the pin and the plate link shall be less than the allowable bearing stress. The bearing area is the pin diameter times the thickness.
5. The pin diameter shall not exceed five times the thickness when the yield stress exceeds 70 ksi (480 MPa).

Appendix A5 Design for Axial Compression

This section applies to members subjected to axial compression caused by forces acting through the centroid of the cross section and assumes failure is by elastic buckling. See Section A8 for members subjected to the combined loading of axial compression and bending.

Appendix A5.1 Gross Area

The compressive normal stress is calculated using the gross area perpendicular to the axis of the loading.

Appendix A5.2 General Stability

General stability shall be provided for the structure as a whole and for each compression member. The critical buckling load for nonstandard loads and support conditions shall be determined by accepted analysis procedures selected by the responsible engineer.

Appendix A5.3 Slenderness Ratio

The slenderness ratio of an axially loaded column is KL/r where K is the effective length factor, L is the column length, and r is the radius of gyration. The effective length factor K can be obtained from tables or calculated using accepted analysis procedures. The radius of gyration is for the axis most likely to fail by buckling. The column length L is the length used in the determination of the effective length factor K. The slenderness ratio of compression members shall not exceed 200. The maximum slenderness ratio for a primary load carrying member should not exceed 120.

Appendix A5.4 Allowable Normal Stress

The allowable normal stress on the gross section of axially loaded compression members is given by

$$F_c = \frac{\left[1 - \frac{1}{2}\left(\frac{KL/r}{C_c}\right)^2\right] F_y}{\left[\frac{5}{3} + \frac{3}{8}\left(\frac{KL/r}{C_c}\right) - \frac{1}{8}\left(\frac{KL/r}{C_c}\right)^3\right]}$$

when the largest effective slenderness ratio of any unbraced element is less than C_c where

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}}$$

The allowable normal stress on the gross section of axially loaded compression members is

$$F_c = \frac{12\pi^2 E}{23(KL/r)^2}$$

when the slenderness ratio exceeds C_c.

Appendix A5.5 Tapered Columns

The allowable compressive stress in tapered columns can be determined using the equations in Section A5.4 provided the effective length factor K has been determined by an accepted analysis procedure. The compressive normal stress f_c shall be calculated using the smallest cross-sectional area in each region with a constant loading.

Appendix A6 Design for Bending

This section applies to straight beams with one or two axes of symmetry, loaded in the vertical plane of symmetry, and having a cross section that is constant or slowly varying with the length. The cross section may be a tube section or a composite section constructed using standard tubes and horizontal plates or vertical plates and other structural shapes. See Section A8 for the design of members subjected to combined loadings involving bending.

Appendix A6.1 Noncompact Square and Rectangular Structural Tubes

The allowable tension and compression stress F_b on the extreme fibers of square and rectangular structural tubes loaded in the plane of their minor axis is $0.60F_y$ provided:

1. The flange width-to-thickness ratio satisfies

$$\frac{b_f\sqrt{F_y}}{238\,t_f} \le 1 \qquad \text{(USCS)}$$

$$\frac{b_f\sqrt{F_y}}{625\,t_f} \le 1 \qquad \text{(SI)}$$

where the flange width b_f is the outside width minus three times the flange thickness t_f.

2. The web thickness-to-depth ratio satisfies

$$\frac{h\sqrt{F_y}}{380\,t_w} \le 1 \qquad \text{(USCS)}$$

$$\frac{h\sqrt{F_y}}{1000\,t_w} \le 1 \qquad \text{(SI)}$$

where h is the outside depth and t_w is the web thickness.

3. The outside depth-to-width ratio is less than six, i.e., $h/6b \le 1$. Lateral bracing is not required for tube sections that satisfy this requirement.

Appendix A6.2 Compact Square and Rectangular Structural Tubes

The allowable tension and compression stress F_b on the extreme fibers of square and rectangular tubes loaded in the plane of their minor axes is $0.66F_y$ provided:

1. The flanges are continuously connected to the webs and have a uniform thickness that is not more than two times the web thickness.

2. The flange width-thickness ratio satisfies

$$\text{USCS:} \quad \frac{b_f\sqrt{F_y}}{190\,t_f} \le 1$$

$$\text{SI:} \quad \frac{b_f\sqrt{F_y}}{499\,t_f} \le 1$$

3. The web depth-thickness ratio satisfies

$$\text{USCS:} \quad \frac{h\sqrt{F_y}}{380\, t_w} \le 1$$

$$\text{SI:} \quad \frac{h\sqrt{F_y}}{1000\, t_w} \le 1$$

where h is the outside depth.

4. An overall depth shall not be greater than six times the flange width b_f.

5. The beam shall be laterally supported at intervals L not exceeding

$$\text{USCS:} \quad \frac{L}{b_f} = \frac{1200(1.625 + M_1\ /M_2)}{F_y}$$

$$\text{SI:} \quad \frac{L}{b_f} = \frac{8270(1.625 + M_1/M_2)}{F_y}$$

except that the lateral spacing need not be less than

$$\text{USCS:} \quad \frac{L}{b_f} = \frac{1200}{F_y}$$

$$\text{SI:} \quad \frac{L}{b_f} = \frac{8270}{F_y}$$

The quantities M_1 and M_2 are the smaller and larger bending moments at the ends of the unbraced length. The end-moment ratio, M_1/M_2, is positive when the beam segment is in reverse (double) curvature bending and negative when it is in single curvature bending.

Appendix A6.3 Light Square and Rectangular Tubes

The allowable tension and compressive stress F_b on extreme fibers of light square and rectangular tubes shall be $0.60F_y$ provided these tubes satisfy the dimension requirements of Section A6.1. The allowable tension and compressive stress on extreme fibers of light square and rectangular tubes with a diameter-thickness ratio greater than those given above shall be 0.60 F_y provided the lateral support and local buckling requirements are determined by special analysis.

Appendix A6.4 Circular Tubes

The allowable tension and compression stress F_b on the extreme fibers of circular tubes is $0.66F_y$ provided the outside diameter-thickness ratio satisfies

$$\text{USCS:} \quad \frac{D}{t} \le \frac{3300}{F_y}$$

$$\text{SI:} \quad \frac{D}{t} \le \frac{22750}{F_y}$$

The allowable tension and compressive stress on extreme fibers of circular tubes with a diameter-thickness ratio greater than those given above shall be $0.60F_y$ provided the lateral support and local buckling requirements are determined by special analysis.

Appendix A6.5 Solid Round and Rectangular Sections

The allowable tension and compression stress F_b on extreme fibers of solid round bars, solid square bars, and solid rectangular bars bent about their weak axis is $0.75F_y$ while the allowable tension and compression stress on extreme fibers of solid rectangular bars bent about their strong axis is $0.60F_y$. Solid rectangular bars bend about their strong axis shall be laterally supported at intervals not exceeding

$$\frac{L}{b} = 1.25\left(\frac{E}{F_y}\right)\left(\frac{b}{h}\right)^2$$

where h is the largest bar dimension and b is the smallest bar dimension. Values of L/b beyond those given by this equation may be used if a lateral stability analysis is performed.

Appendix A6.6 Shear Stress

The allowable shear stress F_v in tubes and solid sections resulting from a bending load is $0.40F_y$ provided the tubes and solid sections satisfy the appropriate requirements for the normal stress. The maximum shear stress shall be calculated at the neutral axis using

$$f_v = \frac{V}{S_v}$$

where S_v is a section property. The correct value of S_v shall be used for circular tubes and all bars. An approximate value given by

$$S_v = 2A_w / 3$$

can be used for tube sections. The web area A_w shall be calculated using the inside height dimension and the total wall thickness. The maximum shear stress shall be recalculated using the correct value for S_v when the approximate value is close to the allowable value.

Appendix A6.7 Composite Beams

Composite beams can be tube sections with top and bottom plates, a tube with side plates, members made using vertical plates and tubes or vertical plates and channels, and box sections constructed using four pieces of plate steel. All composite sections shall be tubular in shape and designed using the noncompact criteria of Section A6.1 and the shear stress shall satisfy the requirements in Section A6.6.

Composite sections composed of tubes with side plates, members made using vertical plates and tubes or vertical plates and channels, and box sections shall have continuous fillet welds. The top and bottom plates can be attached to tube sections with intermittent fillet welds. The top and bottom plates shall extend beyond the critical section one and one-half times their width or a minimum of four inches (100 mm). The plates shall be tapered from one-half their width to the full width over the length of the extension. The plates shall be continuously welded on the three sides of the extension.

Appendix A7 Design for Torsion

This section covers straight members with constant cross sections subjected to a torsion (twisting) loading about their long axis. See Section A8 for members subjected to the combined loading of torsion and bending.

The maximum allowable shear stress F_τ in tube sections and bars is $0.40F_y$. The shear stress is calculated using

$$f_v = M_x / J_\tau$$

where M_x is the twisting moment and J_τ is a section property.

Appendix A8 Design for Combined Loadings

This section covers the design of members subjected to the combined loadings of axial tension and bending, axial compression and bending, and torsion and bending.

Appendix A8.1 Axial Tension and Bending

Solid and tubular members with two axes of symmetry subjected to both axial tension and bending stresses shall be proportioned at all points along their length to satisfy

$$\frac{f_t}{F_t} + \frac{f_b}{F_b} \le 1$$

The calculated bending stress in compression must satisfy

$$\frac{f_{bc}}{F_b} \le 1$$

The allowable tension stress F_t shall satisfy the requirements of Section A4.

The allowable bending stress F_b can be for a compact or noncompact section. The computed shear stress in bending shall satisfy the requirements in Section A6.5 while the requirements of A6.6 shall be satisfied for composite sections.

Appendix A8.2 Axial Compression and Bending

Solid and tubular members with two axes of symmetry subjected to both axial compression and bending stresses shall be proportioned to satisfy

$$\frac{f_c}{F_c} + \frac{f_{bc}}{F_b} \le 1$$

and

$$\frac{f_c}{F_c} \le 0.15$$

The allowable compressive stress F_c is evaluated using the requirements in Section A5. The allowable bending stress F_b can be for a compact or noncompact section. The computed shear stress in bending shall satisfy the requirements in Section A6.5 while the requirements of A6.6 shall be satisfied for composite sections.

Appendix A8.3 Torsion and Bending

Circular and rectangular tubes and solid circular sections subjected to combined loadings of torsion and bending shall be designed on the basis of the maximum shear stress. The allowable shear stress F_τ shall be $0.40F_y$ and the maximum shear stress f_τ shall be calculated using

$$f_\tau = \sqrt{\left(\frac{M_z}{2S_{b1}}\right)^2 + \left(\frac{M_x}{J_\tau}\right)^2}$$

The section shall also satisfy

$$\frac{f_{bc}}{F_b} \le 1 \quad \text{and} \quad \frac{f_{bt}}{F_b} \le 1$$

The allowable bending stress F_b can be for a compact or noncompact section. The computed shear stress in bending shall satisfy the requirements in Section A6.5 while the requirements of A6.6 shall be satisfied for composite sections.

Appendix A9 Design of Fillet Welds

All fillet welds shall be made using E60 or E70 welding rods that conform to the provisions of the American Welding Society *Structural Welding Code—Steel*. All fillet welds shall be designed to fail in shear. The allowable shear stress in a fillet weld is $0.30F_u$ where F_u is the ultimate tensile strength of the rod.

The *minimum size fillet welds* shall be as given in Appendix Table A9.1. Minimum weld size is dependent on the thicker of the two parts, except that the weld size need not exceed the thickness of the thinner part. Weld sizes larger than the thinner part joined are permitted if required by calculated strength. A 1/8 inch (3 mm) fillet weld is considered the minimum practical size.

The *maximum size fillet weld* that is permitted along edges of connected parts shall be:

1. Not greater than the thickness of the material for material less than 1/4 inch (6 mm) thick
2. Not greater than the thickness of the material minus 1/16 inch (1.5 mm) for material 1/4 inch (6 mm) or more in thickness.

Fillet welds greater than 5/16 inch (7.5 mm) must be made with two or more passes of the electrode.

The effective length of a fillet weld is the specified weld length minus twice the weld size. The effective length shall be used for strength calculations. The minimum effective length of fillet welds shall not be less than four times the nominal size, or else the size of the weld shall be considered not to exceed 1/4 of its effective length. The full length of an end or side can be used as the weld length in strength calculations when the weld is turned around a corner for a distance of not less than twice the nominal weld size. Welds should be turned around corners whenever practicable.

Intermittent fillet welds are permitted in composite beams consisting of tubes with top and bottom plates when the required strength is less than that developed by a continuous fillet weld of the smallest practical size. The effective length of any segment of intermittent fillet welding shall be not less than 1 1/2 inch (40 mm). An intermittent fillet weld shall cover a minimum of 15% of the length along any single edge.

Appendix Table A9.1
Minimum size for single pass fillet welds.[a]

USCS		Metric	
Thickness of Thicker Part Jointed, in	**Minimum Size Fillet Weld, in**	**Thickness of Thicker Part Jointed, mm**	**Minimum Size Fillet Weld, mm**
To 1/4 inclusive	1/8	To 6 inclusive	4
Over 1/4 to 1/2	3/16	Over 6 to 12	5
Over 1/2 to 3/4	1/4	Over 12 to 19	7
Over 3/4	5/16	Over 19	8

[a] Table from Hall et al. (1961). USCS table information also in AISC ASD (1989). Metric values converted from the USCS information by the author. The metric material thickness values have been rounded down and the weld sizes have been rounded up.