Midwest Plan Service: Structures and Environment Handbook

Topic	Label	Page
Lumber dimensions	Table 201-6	201.5
Nail sizes	Table 205-2	205.2
[1]Portland Cement types	Left column	207.1
Lumber section properties	Table 405-3	405.2
[1]Equivalent fluid density		410.1
[1]Janssen's equation		411.1-2
Beam Formulas		412.2-412.15
Insulation values	Table 631-1	631.4
	Table 631-3	631,6

Table 201-6. Nominal and minimum dressed lumber sizes.
Thicknesses apply to all widths; widths apply to all thicknesses. Dressed sizes are for dry lumber.

Item	Thicknesses Nominal	Thicknesses Dressed	Face widths Nominal	Face widths Dressed
Boards	1	¾ in.	2	1½ in.
	1¼	1	3	2½
	1½	1¼	4	3½
			5	4½
			6	5½
			7	6½
			8	7¼
			9	8¼
			10	9¼
			11	10¼
			12	11¼
			14	13¼
			16	15¼
Dimension	2	1½	2	1½
	2½	2	3	2½
	3	2½	4	3½
	3½	3	5	4½
	4	3½	6	5½
	4½	4	8	7¼
			12	9¼
			12	11¼
Timbers (Dressed green)	5 and thicker	½ off	5 and wider	½ off

Table 205-2. Nail sizes.

"Steel nails" are threaded, hardened steel.

Penny-weight	Length in.	Wire diameter, in. Wire nails	Steel nails	Spikes
4d	1½"	0.098"		
6d	2	0.113	0.120"	
8d	2½	.131	.120	
10d	3	.148	.135	0.192"
12d	3¼	.148	.135	.192
16d	3½	.162	.148	.207
20d	4"	.192"	.177"	.225"
30d	4½	.207	.177	.244
40d	5	.225	.177	.263
50d	5½	.244	.177	.283
60d	6	.262	.177	.283
70d	7"		.207"	
80d	8		.207	
90d	9		.207	
5⁄16	7			.312
3⁄8	8½			.375

207 CONCRETE AS A MATERIAL

Concrete is a mixture of Portland cement, water, air and aggregates. Portland cement is sold in bulk, or in bags of one cubic foot (94 lb). The aggregates provide volume at low cost, composing 66%-78% of the concrete.

Cement and water form a paste that hardens and glues the aggregates together. The quality of concrete is directly related to the binding qualities of this cement paste.

Selection

General properties

Concrete is very durable and resists attack by water, animal manures, chemicals such as fertilizers, and fire. Use high quality concrete around milk, silage, and animal manure.

Concrete is very weak in tension. Its strength in compression depends on the proportions of the mix. The compressive strength is 2 to 5 times that of wood. Most structural uses involve reinforced concrete, which depends on concrete's strength in compression and steel's strength in tension.

Concrete can be finished in either smooth or rough texture. It can be colored with pigments or painted.

Portland cement

Portland cement got its name from the Isle of Portland, near England, where it was first developed. This improvement over an ancient building material is made by burning limestone and clay, adding gypsum, and grinding the result to a fine consistent powder.

A number of variations have been developed for special purposes:

- Type I, normal Portland cement, is the general purpose type, the most common, and is usually furnished unless an alternative is specified.
- Type II is modified to release less heat during curing, and is therefore suitable in mass concrete—heavy retaining walls, or deadmen for suspension bridges. It is moderately high in resistance to sulphates. Type II is replacing Type I as the basic type in some areas.
- Type III is high-early-strength; it is very finely ground and sets very rapidly. It is useful for slip-form construction and for cold weather jobs.
- Type IV is the lowest heat variety, and while suitable for large masses, develops strength relatively slowly. Its primary use is for mass concrete dams and other large volume structures.
- Type V is especially sulphate-resistant and cures more slowly than Type I.
- Types I, II, and III are available as Types IA, IIA, and IIIA. These are air-entrained Portland cements formulated with a compound that releases many tiny bubbles of air during curing. The resulting concrete is highly resistant to frost action and has some increased resistance to salts. Their use is recommended for all outdoor paving and for concrete exposed to animal wastes, even though slightly weaker than Type I, II, and III.

Air-entrained concrete

An air-entraining agent added to the cement produces millions of tiny bubbles in the concrete giving the concrete greater weathering resistance. Air entraining also reduces the strength and increases the concrete workability. Because the air-entraining agent increases the weathering resistance compared to its reduction of strength, air entraining is usually recommended for all concrete subjected to freezing and thawing. Table 207-1 relates the amounts of air entraining to aggregate size.

Other additives include pigments for coloring, gypsum to retard setting time, calcium to reduce setting time, and bentonite to improve workability.

Table 207-1. Air content for air-entrained concrete.

Max. aggregate size	Amount of air, %
1½", 2", or 2½"	4 to 6
¾" or 1"	5 to 7
⅜" or ½"	6½ to 8½

Water

Cement in concrete cures by the chemical combination of water and cement—not by drying like sheets in the sun. Use water for concrete that is essentially good enough to drink, without harmful chemicals, trash, or organic matter.

The strength of concrete is greatly dependent on the water-cement ratio. Enough water is needed for full curing, but excess water leaves voids when it evaporates. If a mix is too stiff to handle well, do not add water; reject the batch or add cement and water—reduce the amount of aggregates in subsequent batches.

Aggregates

Sand and gravel, the usual aggregates, are glued together by the cured cement paste. They must be free of, or very low in, silt and organic matter, and they must be hard and strong.

Particles up to ¼" are sand, and above ¼" are gravel. Well graded aggregates occupy most of the volume of the concrete, minimizing the amount of cement paste, which is the expensive ingredient. Using spheres for illustration, large balls have less surface area than small ones, requiring less cement. But small balls fit in between the large ones to help fill up the voids.

Table 405-3. Lumber section properties.

Nominal BxD, in.	Dressed BxD, in.	Area A, in^2	Inertia moment I, in^4	Section modulus S, in^3	Shear factor S_v, in^2
1 x 2	¾x1½	1.13	0.21	0.28	0.75
x 3	x2½	1.88	0.98	0.78	1.25
x 4	x3½	2.63	2.68	1.53	1.75
x 5	x4½	3.38	5.70	2.53	2.25
x 6	x5½	4.13	10.40	3.78	2.75
x 7	x6½	4.88	17.16	5.28	3.25
1 x 8	¾x7¼	5.44	23.82	6.57	3.63
x 9	x8¼	6.19	35.09	8.51	4.13
x 10	x9¼	6.94	49.47	10.70	4.63
x 11	x10¼	7.69	67.31	13.13	5.13
x 12	x11¼	8.44	88.99	15.82	5.63
2 x 2	1½x1½	2.25	0.42	0.56	1.50
x 3	x2½	3.75	1.95	1.56	2.50
x 4	x3½	5.25	5.36	3.06	3.50
x 5	x4½	6.75	11.39	5.06	4.50
x 6	x5½	8.25	20.80	7.56	5.50
x 8	x7¼	10.88	47.63	13.14	7.25
2 x 10	1½x9¼	13.88	98.93	21.39	9.25
x 12	x11¼	16.88	177.98	31.64	11.26
3 x 2	2½x1½	3.75	0.70	0.94	2.50
x 3	x2½	6.25	3.26	2.60	4.17
x 4	x3½	8.75	8.93	5.10	5.84
x 5	x4½	11.25	18.98	8.44	7.50
x 6	x5½	13.75	34.66	12.60	9.17
x 8	x7¼	18.13	79.39	21.90	12.09
3 x 10	2½x9¼	23.13	164.89	35.65	15.42
x 12	x11¼	28.13	296.63	52.73	18.76
4 x 2	3½x1½	5.25	0.98	1.31	3.50
x 3	x2½	8.75	4.56	3.65	5.84
x 4	x3½	12.25	12.51	7.15	8.17
x 5	x4½	15.75	26.58	11.81	10.51
x 6	x5½	19.25	48.53	17.65	12.84
x 8	x7¼	x25.38	111.15	30.66	16.93
4 x 10	3½x9¼	32.38	230.84	49.91	21.59
x 12	x11¼	39.38	415.28	73.83	26.26
6 x 2	5½x1½	8.25	1.55	2.06	5.50
x 4	x3½	19.25	19.65	11.23	12.83
x 6	x5½	30.25	76.26	27.73	20.18
x 8	x7½	41.25	193.36	51.56	27.51
x 10	x9½	52.25	392.96	82.73	34.85
x 12	x11½	63.25	697.07	121.23	42.19
8 x 2	7¼x1½	10.88	2.04	2.72	7.25
x 4	x3½	25.38	25.90	14.80	16.92
8 x 6	7½x5½	41.25	103.98	37.81	27.51
x 8	x7½	56.25	263.67	70.31	37.52
x 10	x9½	71.25	535.86	112.81	47.52
x 12	x11½	86.25	950.55	165.31	57.53
10 x 2	9¼x1½	13.88	2.60	3.47	9.25
x 4	x3½	32.38	33.05	18.89	21.58
10 x 6	9½x5½	52.25	131.71	47.90	34.85
x 8	x7½	71.25	333.98	89.06	47.52
x 10	x9½	90.25	678.76	142.90	60.20
x 12	x11½	109.25	1204.03	209.40	72.87
12 x 1	12*x¾	9.00	0.42	1.13	6.00
x 2	x1½	18.00	3.38	4.50	12.01
x 3	x2½	30.00	15.63	12.50	20.01
x 4	x3½	42.00	42.88	24.50	28.01
x 5	x4½	54.00	91.13	40.50	36.02
x 6	x5⁄12	66.00	166.38	60.50	44.02

*Full 12″ wide.

410 EQUIVALENT FLUID DENSITY

Rankine's theory comes from classical soils mechanics. It relates lateral to vertical pressure in noncohesive soils, such as sand and gravel, which are similar in behavior to small grains. It yields adequate equivalent fluid density values for designing shallow grain bin walls. See Chapter 103 for discussion of shallow vs. deep bins.

A companion theory, Coulomb's, analyzes forces on a sliding wedge of material behind a wall. It yields the same values as Rankine's for the angle of the failure plane and the ratio of vertical to lateral pressures when wall friction is neglected.

If a wall restraining the grain moves slightly away from the grain, the lateral pressure decreases, the shear stress increases, and the vertical pressure at the bottom of the grain mass remains unchanged. A plane of shear failure forms and the wedge of grain between it and the wall flows toward the wall as it moves. The failure plane that produces maximum horizontal forces when the pile is level and the wall is vertical is 45° + $\phi/2$ from the horizontal, Fig 410-1.

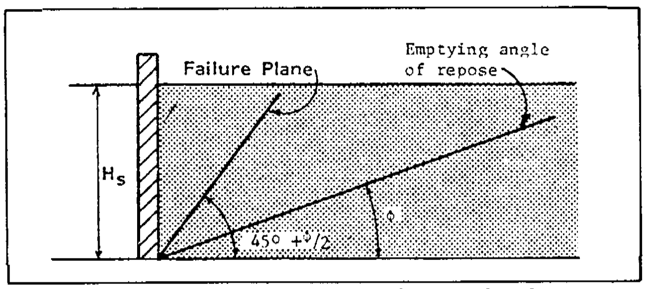


Fig 410-1. Failure plane and emptying angle of repose in grain.

Rankine's definition of the ratio of lateral to vertical pressure for the above condition is:

Eq 410-1.

$$k = \mathrm{Tan}^2(45 - \phi/2) = (1 - \mathrm{Sin}\ \phi)/(1 + \mathrm{Sin}\ \phi)$$

The equivalent fluid density of a grain is:

Eq 410-2.

$$\mathrm{EFD} = (\text{unit weight}) \times k = w \times k$$

Using corn as an example:

Its emptying angle of repose, $\phi = 27°$, is the angle of internal friction.

Therefore:

$k = (1 - \mathrm{Sin}\ 27°)/(1 + \mathrm{Sin}\ 27°) = 0.38$

$\mathrm{EFD} = (48\ \mathrm{lb/ft^3}) \times 0.38 = 18\ \mathrm{lb/ft^3}$

The resulting load diagram on a wall is in Fig 410-2.

The case of an inclined grain surface is more complicated. Lateral pressure is assumed to act at an angle equal to the angle of the coefficient of friction between the wall and the grain mass. If this friction is neglected, which it usually is, then the direction of the thrust is horizontal.

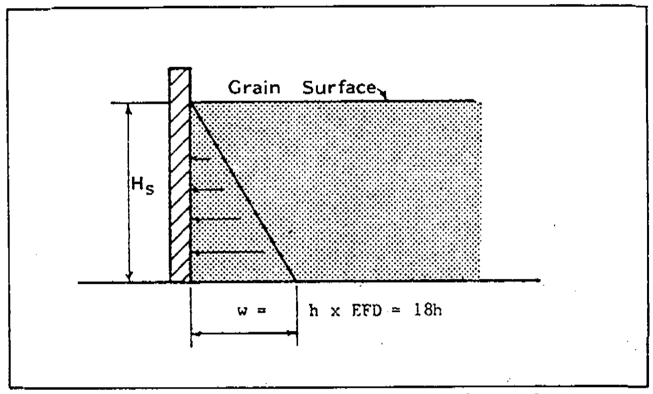


Fig 410-2. Load on a wall from a flat grain surface.

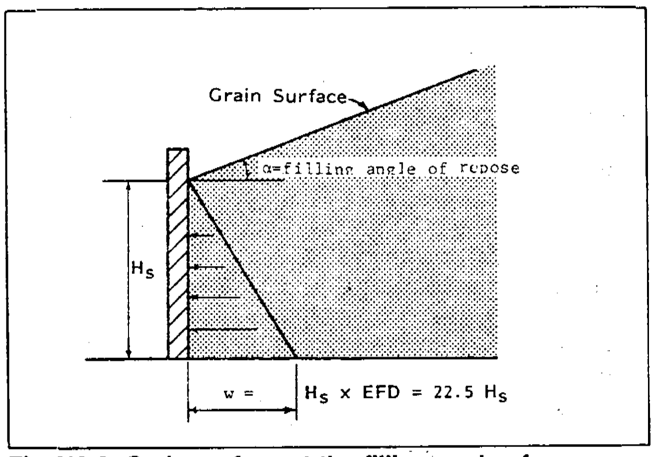


Fig 410-3. Grain surface at the filling angle of repose.

Use the more generalized equation for k if the grain surface or wall is inclined, Fig 410-3.

Eq 410-3.

$$k = \sin^2(\theta-\phi) \div \sin^2(\theta) \times \sin(\theta+\beta) \times T^2$$

$$T = 1 + [\sin(\phi+\beta) \times \sin(\phi-\alpha) \div \sin(\theta-\alpha) \times \sin(\theta+\beta)]^{1/2}$$

k = ratio of lateral to vertical internal pressure

ϕ = angle of internal friction, usually taken as the emptying angle of repose, degrees

θ = angle of wall from horizontal, degrees

β = angle of wall friction, degrees

α = filling angle of repose, degrees

Example, for corn:

$\phi = 27°$

$\theta = 90°$

$\beta = 0°$

$\alpha = 16°$

$T = [(\sin 27\text{-}0)(\sin 27\text{-}16) \div \sin(90\text{-}16)\sin(90+0)]^{1/2} = 1.3$

$k = \sin^2(90\text{-}27)\sin^2(90)\sin(90) \times 1.3^2 = 0.4696$

$\mathrm{EFD} = (48\ \mathrm{lb/ft^3}) \times (0.4696) = 22.5\ \mathrm{lb/ft^3}$

411 JANSSEN'S EQUATION

Many grain bins have significant wall friction with resulting lower vertical and lateral pressures than predicted by equivalent fluid density. The distinction between deep and shallow bins (those with and without significant wall friction) is discussed in Chapter 103.

The interaction of pressures on opposite bin walls has the effect of creating a "pressure dome," Fig 411-1b, which transmits pressure laterally to the sidewalls, creating reactions comparable to those in a structural arch. The lateral bin pressure is accompanied by vertical pressures in the bin walls. The static vertical component is equal to the static lateral pressure times the coefficient of friction between the grain and the bin wall.

Janssen's equation predicts static lateral and static vertical pressures within a grain mass. Because pressures during dynamic or unloading conditions are often greater than static pressures, do not use Janssen's equation directly for bin design. Overpressure factors and other modifications of these static pressures are discussed in Chapter 103.

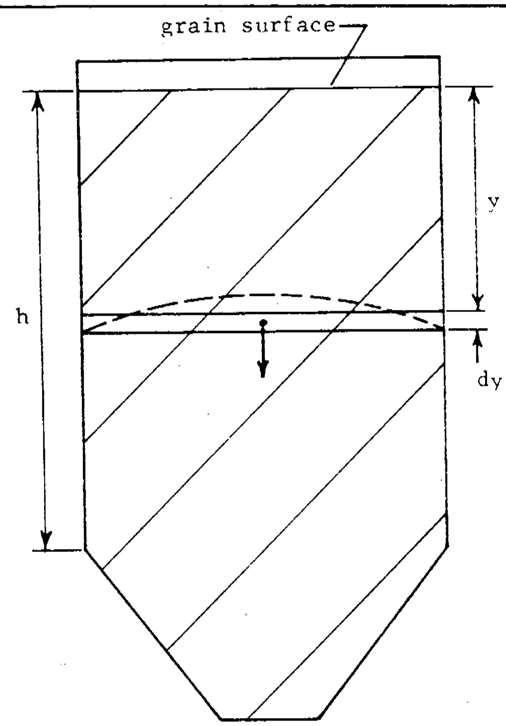
A detailed derivation of Janssen's equation is illustrated in the following material.

Nomenclature:

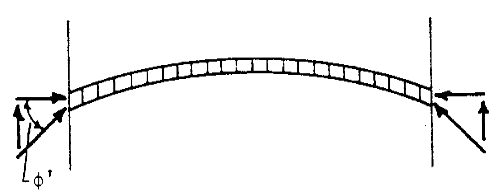
- ϕ = angle of internal friction of the grain (emptying angle of repose)
- β = angle of friction of the grain on the bin walls
- μ' = tan β = coefficient of friction of grain on the bin walls
- w = weight of grain, lb/ft^3
- F = vertical pressure, lb/ft^2
- L_s = static lateral pressure, lb/ft^2
- A = bin area, ft^2
- U = bin circumference, ft
- R = A/U = hydraulic radius of the bin, ft
- H_s = height of the grain, ft
- D = bin diameter, ft

The bin in Fig 411-1a has a uniform area, A, a constant circumference, U, and is filled with grain weighing w lb/ft^3 having an emptying angle of repose, ϕ. Let F be the vertical pressure and L the lateral pressure at any point, with both F and L assumed constant on any horizontal plane.

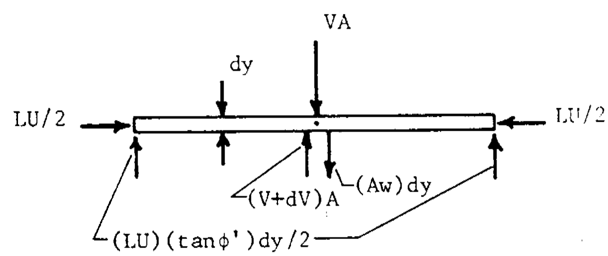
The weight of the grain between the sections of y and y + dy is (Aw)dy; the total frictional force acting upwards at the circumference is (LU)(tan β)dy; the total perpendicular pressure on the upper surface is VA; and total pressure on the lower surface is (F + dF)A.



411-1a. Bin configuration.



411-1b. Arch formation.



411-1c. Free-body diagram.

Fig 411-1. Janssen's equation derivation.

The static vertical pressures must be in equilibrium with their sum equal to zero.

$$0 = FA - (F + dF)A + (Aw)dy - (LU)(\tan \beta)dy$$

Simplify:

$$dF = [w - L(\tan \beta)U/A]dy$$

In a granular mass, the static lateral pressure at any point is equal to the static vertical pressure at that point times a constant for the particular grain, k.

$$L = kV$$

The value of k may be determined by experiment, but the value generally used is the one derived by the Rankine method:

$$k = (1 - \sin \beta)/(1 + \sin \beta)$$

It is an approximation that ignores the friction between the grain and the bin walls.

Also let $R = A/U$ (the hydraulic radius) and $\mu' = \tan \beta$.

Substitute:

$$dF = (w - kF\mu'/R)dy$$

Let:

$$k\mu'/R = n$$

Substitute again:

$$dF = (w - nF)dy, \text{ or:}$$

$$dF/(w - nF) = dy$$

Multiply both sides of the equation by -n and integrate:

$$\ln(w - nF) = -ny + C$$

Evaluate the constant C at $y = 0$ where $F = 0$:

$$C = \ln(w)$$

Rearrange:

$$\ln (w - nF/w) = -ny$$

Take the exponential of each side of the equation:

$$(w - nF)/w = e^{-ny}$$

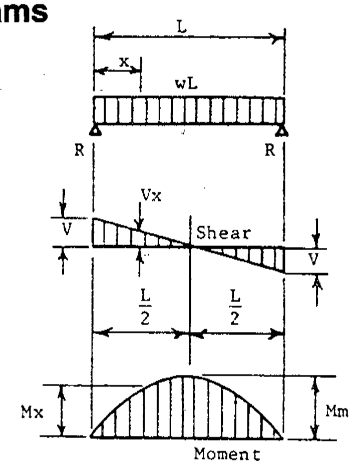
Solve for F:

$$F = w(1 - e^{-ny})/n$$

Substitute for n. The equation becomes:

$$F = (wR/k\mu')(1 - e^{-k\mu' Y/R})$$

Simple Beams



Eq 412-1

$R = V$ . . . . $= \frac{wL}{2}$

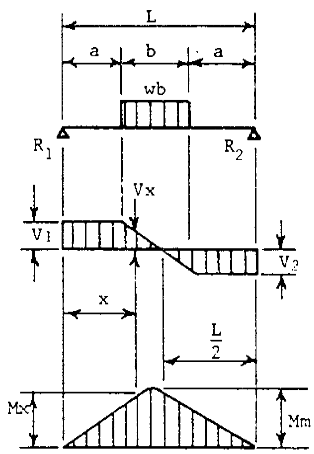
V_x . . . . $= w(\frac{L}{2} - x)$

M_m [@ L/2] . . . . $= \frac{wL^2}{8}$

M_x . . . . $= \frac{wx}{2}(L - x)$

D_m [@ L/2] . . . . $= \frac{5wL^4}{384EI}$

D_x . . . . $= \frac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$



Eq 412-2

$R_1 = R_2 = V_1 = V_2$ . . . . $= \frac{wb}{2}$

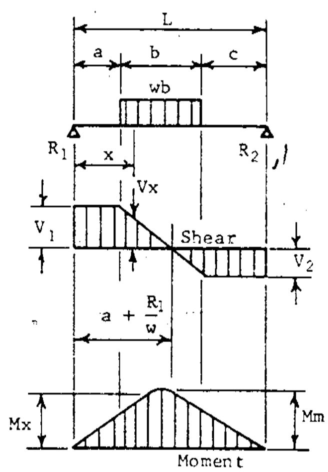
V_x [@ $a < x < (a + b)$] . . . . . $= \frac{wb}{2} - w(x - a)$

M_m [@ L/2] . . . . $= \frac{wb}{8}(2L - b)$

M_x [@ $x < a$] . . . . $= \frac{wbx}{2}$

M_x [@ $a < x < (a + b)$] . . . . . $= \frac{wbx}{2} - \frac{w(x - a)^2}{2}$

D_m [@ $x = L/2$] . . . . $= \frac{5wL^4}{384EI} - \frac{wa^2}{48EI}(3L^2 - 2a^2)$



Eq 412-3

$R_1 = V_1$ [max. @ $a < c$] . . . . $= \frac{wb}{2L}(2c + b)$

$R_2 = V_2$ [max. @ $a > c$] . . . . $= \frac{wb}{2L}(2a + b)$

V_x [@ $a < x < (a + b)$] . . . . $= R_1 - w(x - a)$

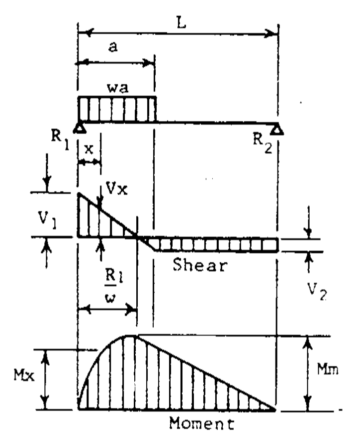
M_m [@ $x = a + \frac{R_1}{w}$] . . . . $= R_1(a + \frac{R_1}{2w})$

M_x [@ $x < a$] . . . . $= R_1x$

M_x [@ $a < x < (a + b)$] . . . . $= R_1x - \frac{w}{2}(x - a)^2$

M_x [@ $(a + b) < x < L$] . . . . $= R_2(L - x)$

D_{c} [when a and c < L/2] . . . $= \frac{5wL^4}{384EI} - \frac{w}{96EI}(3L^2a^2 - 2a^4 + 3L^2c^2 - 2c^4)$



Eq 412-4

$R_1 = V_1max.$ . . . . $= \frac{wa}{2L}(2L - a)$

$R_2 = V_2$ . . . . $= \frac{wa^2}{2L}$

V_x [@ $x < a$] . . . . $= R_1 - wx$

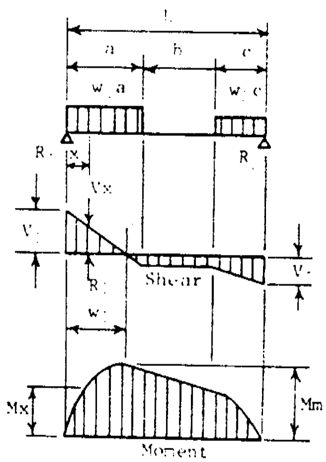
M_m [@ $x = \frac{R_1}{w}$] . . . . $= \frac{R_1^2}{2w}$

M_x [@ $x < a$] . . . . $= R_1x - \frac{wx^2}{2}$

M_x [@ $a < x < L$] . . . . $= R_2(L - x)$

D_x [@ $x < a$] . . . . $= \frac{wx}{24EIL}(a^2(2L - a)^2 - 2ax^2(2L - a) + Lx^3)$

D_x [@ $x > a$] . . . . $= \frac{wa^2(L - x)}{24EIL}(4xL - 2x^2 - a^2)$



Eq 412-5

$R_1 = V_1$ . . . . $= \frac{w_1a(2L - a) + w_2c^2}{2L}$

$R_2 = V_2$ . . . . $= \frac{w_2c(2L - c) + w_1a^2}{2L}$

Vm [@ $w_1a > w_2c$] . . . . $= R_1$

Vm [@ $w_1a < w_2c$] . . . . $= R_2$

Vx [@ $x < a$] . . . . $= R_1 - w_1x$

Mm [@ $x = \frac{R_1}{w_1}$ @ $R_1 < w_1a$] . . . $= \frac{R_1{}^2}{2w_1}$

Mm [@ $x = L - \frac{R_2}{w_2}$ @ $R_2 < w_2c$] . . $= \frac{R_2{}^2}{2w_2}$

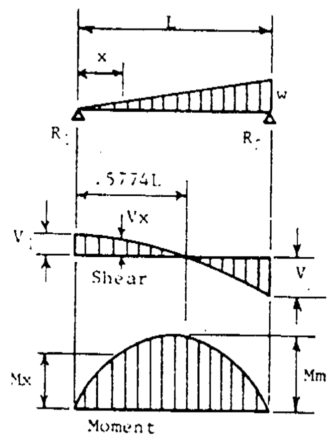
Mx [@ $x < a$] . . . . $= R_1x - \frac{w_1x^2}{2}$

Mx [@ $a < x < (a + b)$] . . . . . $= R_1x - \frac{w_1a}{2}(2x - a)$

Mx [@ $x > (a + b)$] . . . . $= R_2(L - x) - \frac{w_2(L - x)^2}{2}$

D℄ [when a and c < L/2] . . . . $= \frac{w_1a^2}{96EI}(3L^2 - 2a^2)$

$+ \frac{w_2c^2}{96EI}(3L^2 - 2c^2)$



Eq 412-6

W . . . . $= wL/2$

$R_1 = V_1$ . . . . $= wL/6$

$R_2 = V_2max$. . . . . $= wL/3$

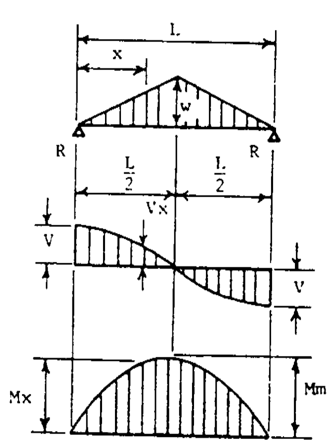
Vx . . . . $= \frac{wL}{6} - \frac{wx^2}{2L}$

Mm [@ x = .5774L] . . . . . . $= \frac{wL^2}{9\sqrt{3}} = 0.0642wL^2$

Mx . . . . $= \frac{wx}{6L}(L^2 - x^2)$

Dm [@ x = .5193L] . . . . . . $= 0.00652\ \frac{wL^4}{EI}$

Dx . . . . $= \frac{wx}{360EIL}\ (3x^4 - 10L^2x^2 + 7L^4)$



Eq 412-7

W . . . . $= wL/2$

R = V . . . . $= wL/4$

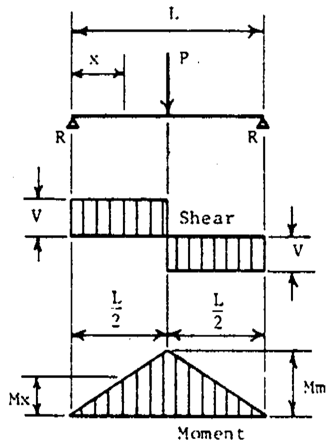
Vx [@ $x < \frac{L}{2}$] . . . . $= \frac{w}{4L}(L^2 - 4x^2)$

Mm [@ center] . . . . $= wL^2/12$

Mx [@ $x < \frac{L}{2}$] . . . . $= \frac{wx}{2}\ (\frac{L}{2} - \frac{2x^2}{3L})$

Dm [@ center] . . . . $= \frac{wL^4}{120EI}$

Dx . . . . $= \frac{wx}{960EIL}(5L^2 - 4x^2)^2$



Eq 412-8

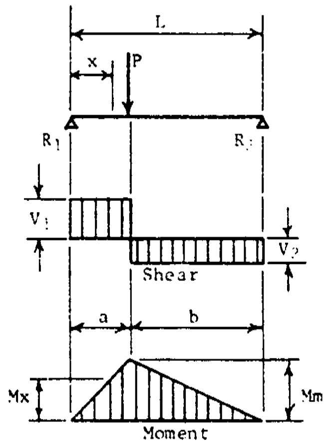
$R = V$ . . . . $= P/2$

M_m [@ load] . . . . $= PL/4$

M_x [@ $x < L/2$] . . . . $= Px/2$

D_m [@ load] . . . . $= \frac{PL^3}{48EI}$

D_x [@ $x < L/2$] . . . . $= \frac{Px}{48EI}(3L^2 - 4x^2)$



Eq 412-9

$R_1 = V_1$ [max. @ $a < b$] . . . . . . $= Pb/L$

$R_2 = V_2$ [max. @ $a > b$] . . . . . . $= Pa/L$

M_m [@ load] . . . . $= Pab/L$

M_x [@ $x < a$] . . . . $= Pbx/L$

M_x [@ $a < x < L$] . . . . $= \frac{Pa}{L}(L - x)$

D_m [@ $x = \sqrt{\frac{a(a + 2b)}{3}}$ @ $a > b$] . . $= \frac{Pab(a + 2b)\sqrt{3a(a + 2b)}}{27\ EIL}$

D_a [@ load] . . . . $= \frac{Pa^2b^2}{3EIL}$

D_x [@ $x < a$] . . . . $= \frac{Pbx}{6EIL}(L^2 - b^2 - x^2)$

D_x [@ $a < x < L$] . . . . $= \frac{Pa\ (L - x)}{6EIL}(-a^2 + 2xL - x^2)$

2 Equal loads

Eq 412-10 a = b = c = L/3

$R = V$ . . . . $= P$

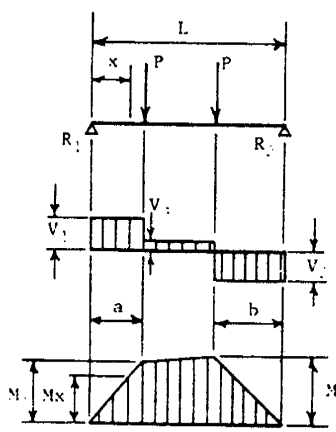
M_m [between loads] . . . . . . $= PL/3$

M_x [@ $x < L/3$] . . . . $= Px$

D_m [@ center] . . . . $= \frac{23PL^3}{648EI}$

D_x [@ $x < a$] . . . . $= \frac{Px}{6EI}\left(\frac{2L^2}{3} - x^2\right)$

D_x [@ $a < x < (L - a)$] . . . $= \frac{PL}{18EI}(3Lx - 3x^2 - L^2/9)$



Eq 412-11 a = b ≠ c

$R = V$ . . . . $= P$

M_m [between loads] . . . . $= Pa$

M_x [@ $x < a$] . . . . $= Px$

D_m [@ center] . . . . $= \frac{Pa}{24EI}(3L^2 - 4a^2)$

D_x [@ $x < a$] . . . . $= \frac{Px}{6EI}(3La - 3a^2 - x^2)$

D_x [@ $a < x < (L - a)$] . . . . . . $= \frac{Pa}{6EI}(3Lx - 3x^2 - a^2)$

Eq 412-12 a ≠ b ≠ c

$R_1 = V_1$ [max. @ $a < b$] . . . . $= \frac{P}{L}(L - a + b)$

$R_2 = V_2$ [max. @ $a > b$] . . . . $= \frac{P}{L}(L - b + a)$

V_3 . . . . $= \frac{P}{L}(b - a)$

M_1 [max. @ $a > b$] . . . . $= R_1a$

M_2 [max. @ $a < b$] . . . . $= R_2b$

M_x [@ $x < a$] . . . . $= R_1x$

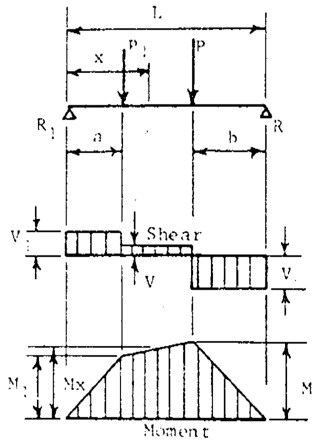
M_x [@ $a < x < (L - b)$] . . . . $= R_1x - P(x - a)$

M_x [@ $(L - b) < x < L$] . . . . $= R_2(L - x)$

NOTE: For deflections use superposition of 2 single concentrated loads.

2 Unequal loads

Eq 412-13 $P_1 \neq P_2$



$R_1 = V_1$ . . . . $= \frac{P_1(L - a) + P_2 b}{L}$

$R_2 = V_2$ . . . . $= \frac{P_1 a + P_2(L - b)}{L}$

V_3 . . . . $= R_1 - P_1$

M_1 [max. @ $R_1 < P_1$] . . . $= R_1 a$

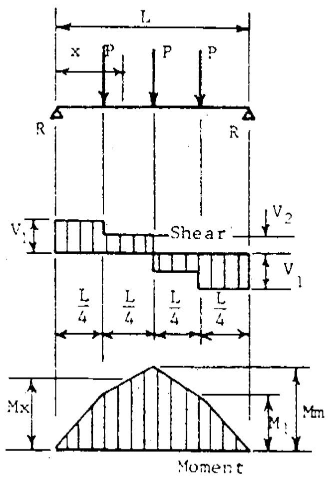
M_2 [max. @ $R_2 < P_2$] . . . $= R_2 b$

M_x [@ $x < a$] . . . . $= R_1 x$

M_x [@ $a < x < (L - b)$] . . $= R_1 x - P_1(x - a)$

NOTE: For deflections use superposition of 2 single concentrated loads.

Eq 412-14



$R = V_1$ . . . . $= 3P/2$

V_2 . . . . $= P/2$

M_x [@ $x < L/4$] . . . . . $= \frac{3Px}{2}$

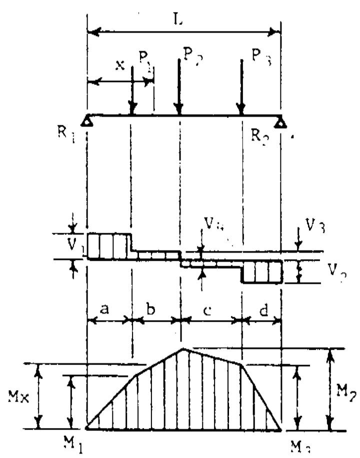
M_x [@ $L/4 < x < L/2$] . . $= \frac{Px}{2} + \frac{PL}{4}$

M_1 [@ $x = L/4$] . . . . . $= \frac{3PL}{8}$

M_m [@ $x = L/2$] . . . . . $= PL/2$

D_m [@ $x = L/2$] . . . . . $= \frac{19PL^3}{384\ EI}$

Eq 412-15



$R_1 = V_1$ . . . . $= \frac{P_1(L - a) + P_2(c + d) + P_3 d}{L}$

$R_2 = V_2$ . . . . $= \frac{P_1 a + P_2(a + b) + P_3(L - d)}{L}$

V_3 . . . . $= R_1 - P_1$

V_4 . . . . $= R_1 - P_1 - P_2$

M_1 . . . . $= R_1 a$

M_2 . . . . $= R_1(a + b) - P_1 b$

M_3 . . . . $= R_1(L - d) - P_1(b + c) - P_2 c$

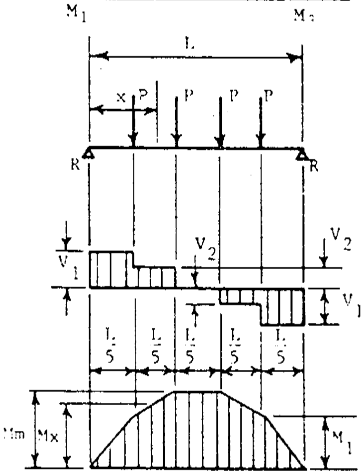
M_m [when $P_1 \geq R_1$] . . . . . . $= M_1$

M_m [when $P_1 + P_2 \geq R_1, P_2 + P_3 \geq R_2$] $= M_2$

M_m [when $P_3 \geq R_2$] . . . . . . $= M_3$

NOTE: For deflections use superposition of 3 single concentrated loads.

Eq 412-16



$R = V_1$ . . . . $= 2P$

V_2 . . . . $= P$

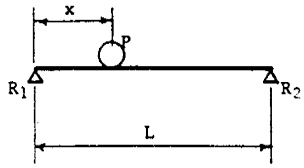
M_x [@ $x < L/5$] . . . . $= 2Px$

M_x [@ $L/5 < x < 2L/5$] . . . . $= Px + PL/5$

M_1 . . . . $= \frac{2PL}{5}$

M_m [@ $x = L/2$] . . . . $= \frac{3PL}{5}$

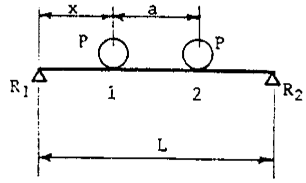
D_x [@ $x = L/2$] . . . . $= \frac{3.78\ PL^3}{60EI}$



Eq 412-17

$R_1m = V_1m$ [@ $x = 0$] . . . . $= P$

Mm [@ load, @ $x = \frac{L}{2}$] . . . $= \frac{PL}{4}$

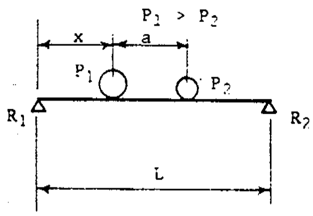


Eq 412-18

$R_1m = V_1m$ [@ $x = 0$] . . . . $= P(2 - \frac{a}{L})$

Mm:

- [@ $a < .586L$; @ load 1 @ $x = \frac{1}{2}(L - \frac{a}{2})$] . . $= \frac{P}{2L}(L - \frac{a}{2})^2$
- [@ $a > .586L$ with one load at center of span] . . $= \frac{PL}{4}$

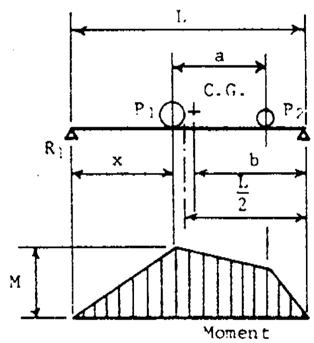


Eq 412-19

$R_1m = V_1m$ [@ $x = 0$] . . . . $= P_1 + P_2\frac{L - a}{L}$

Mm:

- [under P_1, @ $x = \frac{1}{2}(L - \frac{P_2 a}{P_1 + P_2})$] . $= (P_1 + P_2)\frac{x^2}{L}$
- [Mm may be with P_1 at center of span and P_2 off span] . $= \frac{P_1 L}{4}$

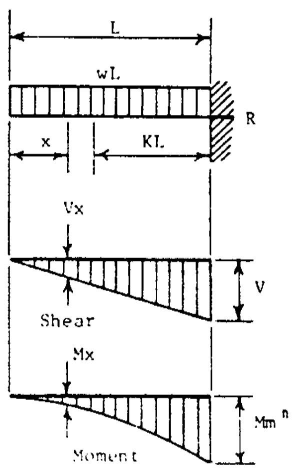


Maximum shear is at one support when one of the loads is at that support. With several moving loads, locate them for maximum shear by trial.

Maximum moment is under one of the loads when that load is as far from one support as the center of gravity of all the moving loads on the beam is from the other support.

Mm is at P_1 when $x = b$, and when the span center line is midway between the center of gravity of loads and the nearest concentrated load.

Cantilever Beams



Eq 412-20

$R = V$ . . . . $= wL$

Vx . . . . $= wx$

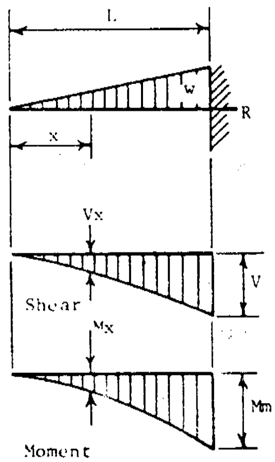
Mm [@ fixed end] . . . . $= \frac{-wL^2}{2}$

Mx . . . . $= \frac{-wx^2}{2}$

Dm [@ free end] . . . . $= \frac{wL^4}{8EI}$

Dx . . . . $= \frac{w}{24EI}(x^4 - 4L^3x + 3L^4)$

For partial load from support to distance KL, substitute KL for L. Measure x from left end of KL. Dm then becomes D_{KL}.



Eq 412-21

W . . . . = $wL/2$

R = V . . . . = $wL/2$

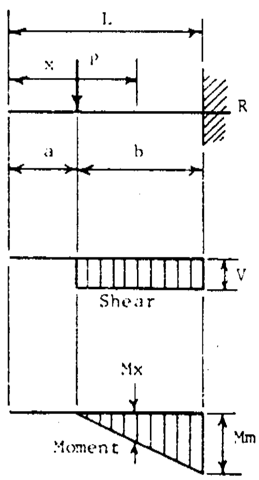
Vx . . . . = $wx^2/2L$

Mm [@ fixed end] . . . . = $-\,wL^2/6$

Mx . . . . = $-\,wx^3/6L$

Dm [@ free end] . . . . = $\frac{wL^4}{30EI}$

Dx . . . . = $\frac{w}{120EIL}(x^5 - 5L^4x + 4L^5)$



Eq 412-22 a = 0 (load at free end)

R = V . . . . = P

Mm [@ fixed end] . . . . = $-\,PL$

Mx . . . . = $-\,Px$

Dm [@ free end] . . . . = $\frac{PL^3}{3EI}$

Dx . . . . = $\frac{P}{6EI}(2L^3 - 3L^2x + x^3)$

Eq 412-23 a ≠ 0

R = V [@ a < x < L] . . . . . . = P

Mm [@ fixed end] . . . . = $-\,Pb$

Mx [@ x > a] . . . . = $-\,P(x - a)$

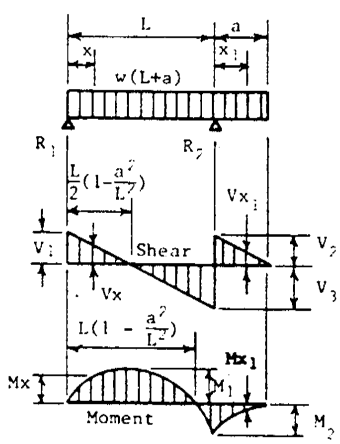
Dm [@ free end] . . . . = $\frac{Pb^2}{6EI}(3L - b)$

Da [@ load] . . . . = $\frac{Pb^3}{3EI}$

Dx [@ x < a] . . . . = $\frac{Pb^2}{6EI}(3L - 3x - b)$

Dx [@ x > a] . . . . = $\frac{P(L - x)^2}{6EI}(3b - L + x)$

Overhang Beams



Eq 412-24

$R_1 = V_1$ . . . . = $\frac{w}{2L}(L^2 - a^2)$

$R_2 = V_2 + V_3$ . . . . = $\frac{w}{2L}(L + a)^2$

V_2 . . . . = wa

V_3 . . . . = $\frac{w}{2L}(L^2 + a^2)$

Vx . . . . = $R_1 - wx$

Vx_1 . . . . = $w(a - x_1)$

M_1 [@ $x = \frac{L}{2}(1 - \frac{a^2}{L^2})$] . . . . . . = $\frac{w}{8L^2}(L + a)^2(L - a)^2$

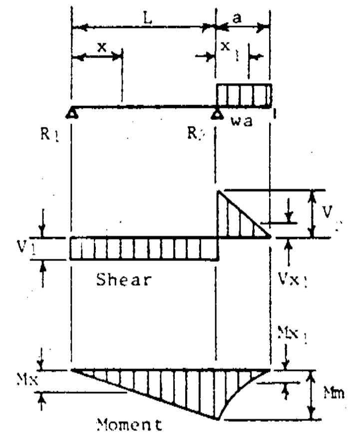
M_2 [@ R_2] . . . . = $\frac{-\,wa^2}{2}$

Mx . . . . = $\frac{wx}{2L}(L^2 - a^2 - xL)$

Mx_1 . . . . = $-\,\frac{w}{2}(a - x_1)^2$

Dx . . . . . = $\frac{wx}{24EIL}(L^4 - 2L^2x^2 + Lx^3 - 2a^2L^2 + 2a^2x^2)$

Dx_1 . . . . . = $\frac{wx_1}{24EI}(4a^2L - L^3 + 6a^2x_1 - 4ax_1^2 + x_1^3)$



Eq 412-25

$R_1 = V_1 \ldots\ldots = -\frac{wa^2}{2L}$

$R_2 = V_1 + V_2 \ldots\ldots = \frac{wa}{2L}(2L + a)$

$V_2 \ldots\ldots = wa$

$Vx_1 \ldots\ldots = w(a - x_1)$

Mm [@ R_2] $\ldots\ldots = \frac{-wa^2}{2}$

$Mx \ldots\ldots = \frac{-wa^2x}{2L}$

$Mx_1 \ldots\ldots = -\frac{w}{2}(a - x_1)^2$

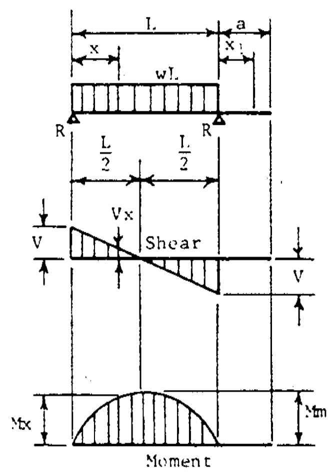
Dmx [@ $x = \frac{L}{3}$] $\ldots\ldots = -\frac{wa^2L^2}{18\sqrt{3}\,EI}$

$= -.03208\frac{wa^2L^2}{EI}$

Dmx_1 [@ $x_1 = a$] $\ldots\ldots = \frac{wa^3}{24EI}(4L + 3a)$

$Dx \ldots\ldots = -\frac{wa^2x}{12EIL}(L^2 - x^2)$

$Dx_1 \ldots\ldots = \frac{wx_1}{24EI}(4a^2L + 6a^2x_1 - 4ax_1^2 + x_1^3)$



Eq 412-26

$R = V \ldots\ldots = \frac{wL}{2}$

$Vx \ldots\ldots = w(\frac{L}{2} - x)$

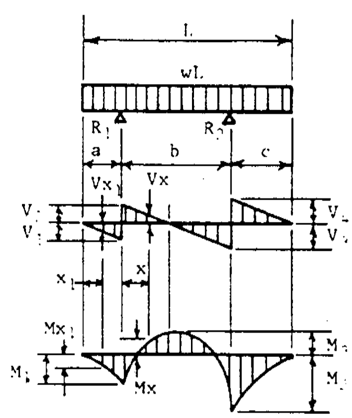
Mm [@ center] $\ldots\ldots = \frac{wL^2}{8}$

$Mx \ldots\ldots = \frac{wx}{2}(L - x)$

Dm [@ center] $\ldots\ldots = \frac{5wL^4}{384EI}$

$Dx \ldots\ldots = \frac{wx}{24EI}(L^3 - 2Lx^2 + x^3)$

$Dx_1 \ldots\ldots = -\frac{wL^3x_1}{24EI}$



Eq 412-27

$R_1 \ldots\ldots = \frac{wL(L - 2c)}{2b}$

$R_2 \ldots\ldots = \frac{wL(L - 2a)}{2b}$

$V_1 \ldots\ldots = wa$

$V_2 \ldots\ldots = R_1 - V_1$

$V_3 \ldots\ldots = R_2 - V_4$

$V_4 \ldots\ldots = wc$

$Vx_1 \ldots\ldots = V_1 - wx_1$

Vx [@ $x < L$] $\ldots\ldots = R_1 - w(a + x_1)$

Vm [@ $a < c$] $\ldots\ldots = R_2 - wc$

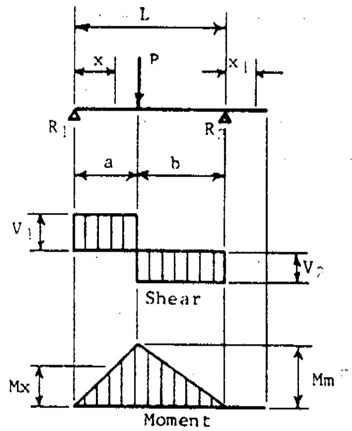
$M_1 \ldots\ldots = -\frac{wa^2}{2}$

$M_2 \ldots\ldots = -\frac{wc^2}{2}$

$M_3 \ldots\ldots = R_1(\frac{R_1}{2w} - a)$

Mx [max. @ $x = \frac{R_1}{w} - a$] $\ldots\ldots = R_1x - \frac{w(a + x)^2}{2}$

$Mx_1 \ldots\ldots = \frac{-wx_1^2}{2}$



Eq 412-28

$R_1 = V_1$ [max. @ $a < b$] . . . . $= \frac{Pb}{L}$

$R_2 = V_2$ [max. @ $a > b$] . . . . $= \frac{Pa}{L}$

Mm [@ load] . . . . $= \frac{Pab}{L}$

Mx [@ $x < a$] . . . . $= \frac{Pbx}{L}$

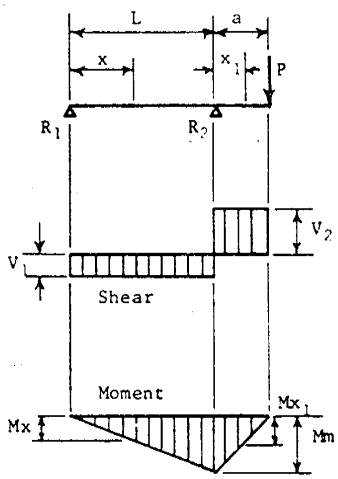
Dm [@ $x = \sqrt{\frac{a(a+2b)}{3}}$ @ $a > b$] $= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIL}$

Da [@ load] . . . . $= \frac{Pa^2b^2}{3EIL}$

Dx [@ $x < a$] . . . . $= \frac{Pbx}{6EIL}(L^2 - b^2 - x^2)$

Dx [@ $x > a$] . . . . $= \frac{Pa(L-x)}{6EIL}(2Lx - x^2 - a^2)$

Dx_1 . . . . $= \frac{Pabx_1}{6EIL}(L + a)$



Eq 412-29

$R_1 = V_1$ . . . . $= \frac{-Pa}{L}$

$R_2 = V_1 + V_2$ . . . . $= \frac{P}{L}(L + a)$

V_2 . . . . $= P$

Mm [@ R_2] . . . . $= -Pa$

Mx [between supports] . . . . $= \frac{-Pax}{L}$

Mx_1 [for overhang] . . . . . . $= -P(a - x_1)$

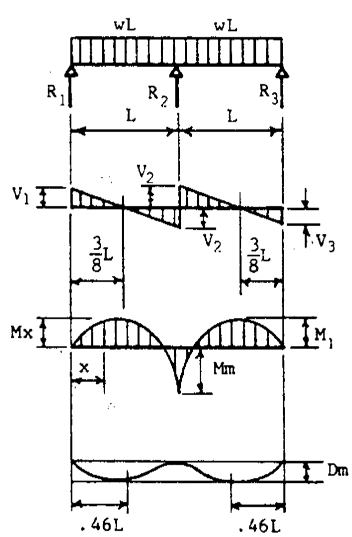
Dm [@ $x = \frac{L}{\sqrt{3}}$] . . . . $= -.06415\frac{PaL^2}{EI}$

Dm [@ $x_1 = a$] . . . . $= \frac{Pa^2}{3EI}(L + a)$

Dx [between supports] . . . . $= \frac{-Pax}{6EIL}(L^2 - x^2)$

Dx_1 [for overhang] . . . . . . $= \frac{Px_1}{6EI}(2aL + 3ax_1 - x_1^2)$

Multi-Span Beams



Eq 412-30

$R_1 = V_1 = R_3 = V_3$ . . . . $= \frac{3wL}{8}$

R_2 . . . . $= \frac{10wL}{8}$

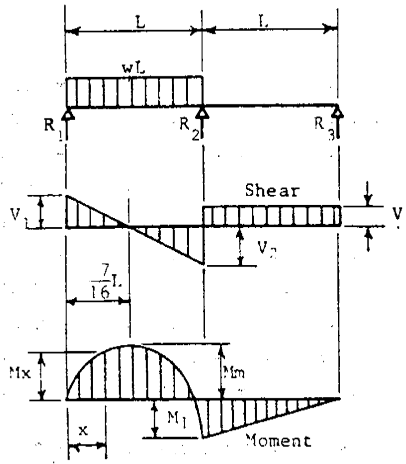
$V_2 = Vm$ . . . . $= \frac{5wL}{8}$

Mm . . . . $= -\frac{wL^2}{8}$

M_1 [@ $x = \frac{3L}{8}$] . . . . $= \frac{9wL^2}{128}$

Mx [@ $x < L$] . . . . $= \frac{3wLx}{8} - \frac{wx^2}{2}$

Dm [@ x = approx. 0.46L] . . . . $= \frac{wL^4}{185EI}$



Eq 412-31

$R_1 = V_1$ . . . . $= \frac{7}{16}wL$

$R_2 = V_2 + V_3$ . . . . $= \frac{5}{8}wL$

$R_3 = V_3$ . . . . $= -\frac{1}{16}wL$

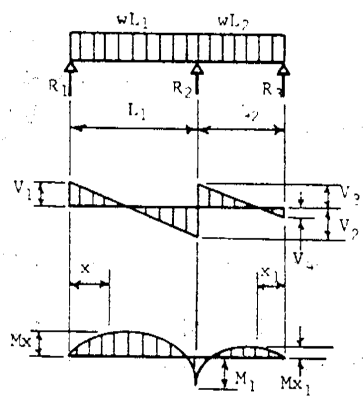
V_2 . . . . $= \frac{9}{16}wL$

Mm [@ $x = \frac{7}{16}L$] . . . . $= \frac{49}{512}wL^2$

M_1 [@ R_2] . . . . $= -\frac{1}{16}wL^2$

Mx [@ $x < L$] . . . . $= \frac{wx}{16}(7L - 8x)$

Dm [@ $x = 0.472L$] . . . . $= \frac{0.0092wL^4}{EI}$



Eq 412-32

$R_1 = V_1$ . . . . $= \frac{M_1}{L_1} + \frac{wL_1}{2}$

R_2 . . . . $= wL_1 + wL_2 - R_1 - R_3$

$R_3 = V_4$ . . . . $= \frac{M_1}{L_2} + \frac{wL_2}{2}$

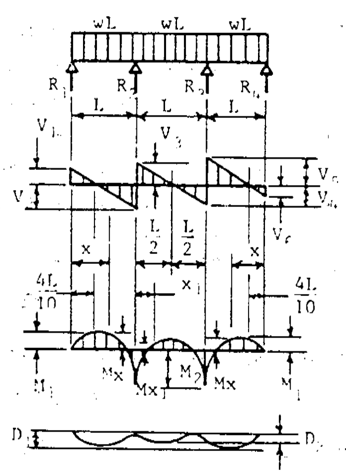
V_2 . . . . $= wL_1 - R_1$

V_3 . . . . $= wL_2 - R_3$

M_1 . . . . $= -\frac{wL_2{}^3 + wL_1{}^3}{8(L_1 + L_2)}$

Mx [@ $x < L_1$ max. @ $x = \frac{R_1}{w}$] . . $= R_1x = \frac{wx^2}{2}$

Mx_1 [@ $x_1 < L_2$ max. @ $x_1 = \frac{R_3}{w}$] $= R_3x_1 - \frac{wx_1{}^2}{2}$



Max. Pos. Mom. M_1 [@ $x = \frac{4L}{10}$] $= \frac{2wL^2}{25}$

Max. Neg. Mom. M_2 [@ $x = L$] $= -\frac{wL^2}{10}$

Eq 412-33

$R_1 = R_4 = V_1 = V_6$ . . . . $= \frac{4wL}{10}$

$R_2 = R_3$ . . . . $= \frac{11wL}{10}$

$V_2 = V_5$ . . . . $= \frac{6wL}{10}$

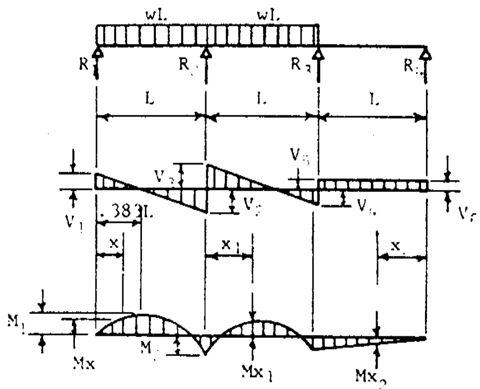
$V_3 = V_4$ . . . . $= \frac{wL}{2}$

Mx [@ $x < L$ max. @ $x = \frac{4L}{10}$] . . . . . $= \frac{4wLx}{10} - \frac{wx^2}{2}$

Mx_1 [@ $x_1 < L$ max. @ $x_1 = L/2$] . . . $= \frac{wLx_1}{2} - \frac{wL^2}{10} - \frac{wx_1{}^2}{2}$

$D_1 = Dm$ . . . . $= \frac{4wL^4}{581EI}$

D_2 . . . . $= \frac{wL^4}{1920EI}$



Eq 412-34

$R_1 = V_1$ . . . . = 0.383wL

R_2 . . . . = 1.20wL

R_3 . . . . = 0.450wL

R_4 . . . . = −0.033wL

V_2 . . . . = 0.617wL

V_3 . . . . = 0.583wL

V_4 . . . . = 0.417wL

$V_5 = V_6$ . . . . = 0.033wL

Max. Neg. Mom. M_2 [@ x = L] . . . . . = $-0.1167wL^2$

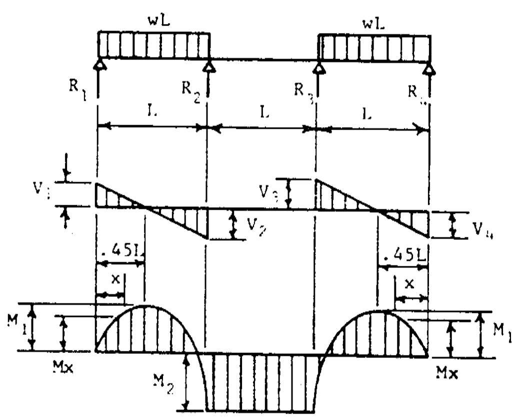
Max. Pos. Mom. M_1 [@ x = 0.383L] . . = $0.0735wL^2$

Mx [@ x < L max. @ x = 0.383L] . . . = $0.383wLx - \frac{wx^2}{2}$

Mx_1 [@ x_1 < L max. @ x_1 = 0.583L] . . = $0.583wLx_1 - 0.1167wL^2 - \frac{wx_1^2}{2}$

Mx_2 [@ x_2 < L] . . . . = $-0.033wLx_2$

Dm [@ x = 0.430L] . . . . = $\frac{0.0059wL^4}{EI}$



Eq 412-35

$R_1 = R_4 = V_1 = V_4$ . . . . = 0.450wL

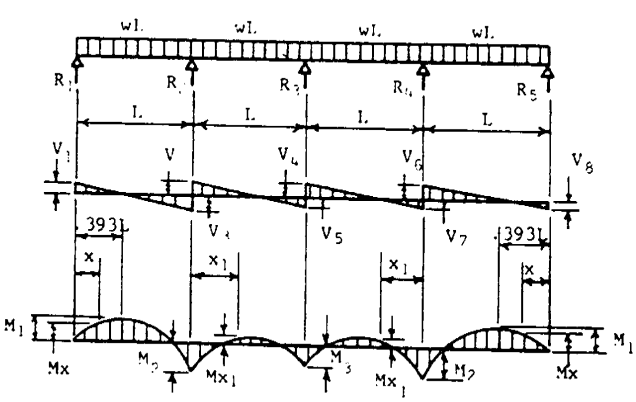
$R_2 = R_3 = V_2 = V_3$ . . . . = 0.550wL

Max. Neg. Mom. M_2 [@ x = L] . . . . . = $-0.05wL^2$

Max. Pos. Mom. M_1 [@ x = 0.450L] . = $0.1013wL^2$

Mx [@ x < L] . . . . = $0.450wLx - \frac{wx^2}{2}$

Dm [@ x = 0.479L] . . . . = $\frac{0.0099wL^4}{EI}$



Max. Pos. Mom. M_1 [@ x = 0.393L] . . = $0.0772wL^2$

Max. Neg. Mom. M_2 [@ x = L] . . . . . = $-0.1071wL^2$

Eq 412-36

$R_1 = R_5 = V_1 = V_8$ . . . . = 0.393wL

$R_2 = R_4$ . . . . = 1.143wL

R_3 . . . . = 0.928wL

$V_2 = V_7$ . . . . = 0.536wL

$V_3 = V_6$ . . . . = 0.607wL

$V_4 = V_5$ . . . . = 0.464wL

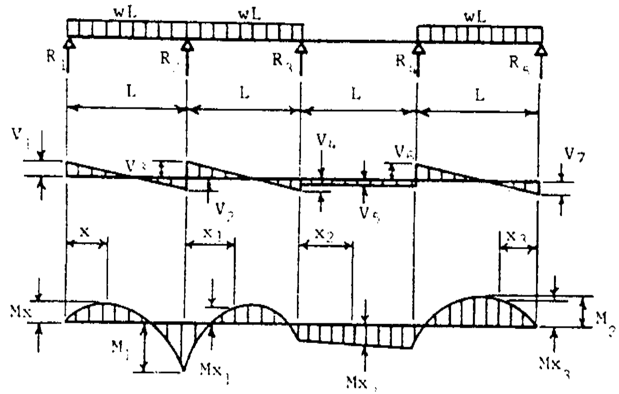
M_3 . . . . = $-0.0714wL^2$

Mx [@ x < L max. @ x = 0.393L] . . . . . = $0.393wLx - \frac{wx^2}{2}$

Mx_1 [@ x_1 < L max. @ x_1 = 0.536L] . . = $0.536wLx_1 - 0.1071wL^2 - \frac{wx_1^2}{2}$

Dm [@ x = 0.440L] . . . . = $\frac{0.0065wL^4}{EI}$

$M_4 = 0.036wL^2$



Max. Pos. Mom. M_2 [@ $x_3 = 0.442L$] = $0.0977wL^2$

Max. Neg. Mom. M_1 [@ $x = L$] . . . = $-0.1205wL^2$

Eq 412-37

$R_1 = V_1$ . . . . = $0.380wL$

R_2 . . . . = $1.223wL$

R_3 . . . . = $0.357wL$

R_4 . . . . = $0.598wL$

$R_5 = V_7$ . . . . = $0.442wL$

V_2 . . . . = $0.620wL$

V_3 . . . . = $0.603wL$

V_4 . . . . = $0.397wL$

V_5 . . . . = $0.040wL$

V_6 . . . . = $0.558wL$

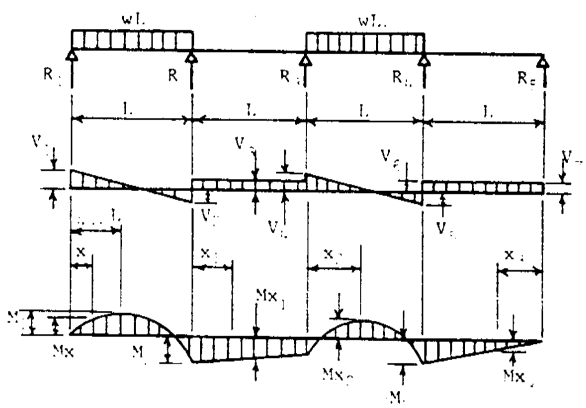
Mx [@ $x < L$ max. @ $x = 0.380L$] . . . = $0.380wLx - \frac{wx^2}{2}$

Mx_1 [@ $x_1 < L$ max. @ $x_1 = 0.603L$] . = $0.603wLx_1 - 0.1205wL^2 - \frac{wx_1^2}{2}$

Mx_2 [@ $x_2 < L$ max. @ $x_2 = L$] . . . = $-0.04wLx_2 - 0.0179wL^2$

Mx_3 [@ $x_3 < L$ max. @ $x_3 = 0.442L$] . = $0.442wLx_3 - \frac{wx_3^2}{2}$

Dm [@ $x = 0.475L$] . . . . = $\frac{0.0094wL^4}{EI}$



Max. Pos. Mom. M_1 [@ $x = 0.446L$] . = $0.0996wL^2$

Max. Neg. Mom. M_2 [@ $x = L$ and @ $x_2 = L$] . . . . = $-0.0536wL^2$

Eq 412-38

$R_1 = V_1$ . . . . = $0.446wL$

$R_2 = R_4$ . . . . = $0.572wL$

R_3 . . . . = $0.464wL$

R_5 . . . . = $-0.054wL$

V_2 . . . . = $0.554wL$

V_3 . . . . = $0.018wL$

V_4 . . . . = $0.482wL$

V_5 . . . . = $0.518wL$

$V_6 = V_7$ . . . . = $0.054wL$

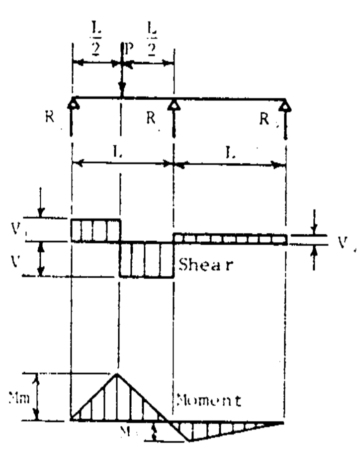
Mx [@ $x < L$ max. @ $x = 0.446L$] . . . = $0.446wLx - \frac{wx^2}{2}$

Mx_1 [@ $x_1 < L$ max. @ $x_1 = 0$] . . . = $0.018wLx_1 - 0.0536wL^2$

Mx_2 [@ $x_2 < L$ max. @ $x_2 = 0.482L$] . = $0.482wLx_2 - 0.0357wL^2 - \frac{wx_2^2}{2}$

Mx_3 [@ $x_3 < L$ max. @ $x_3 = L$] . . . = $-0.054wLx_3$

Dm [@ $x = 0.477L$] . . . . = $\frac{0.0097wL^4}{EI}$



Eq 412-39

$R_1 = V_1$ . . . . = $\frac{13}{32}P$

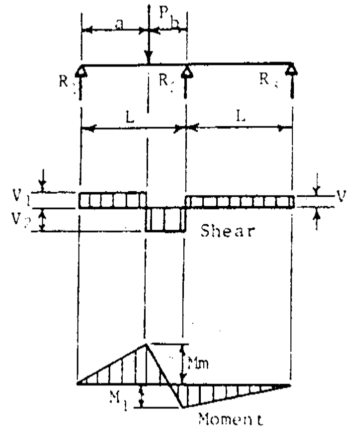
$R_2 = V_2 + V_3$ . . . . = $\frac{11}{16}P$

$R_3 = V_3$ . . . . = $-\frac{3}{32}P$

V_2 . . . . = $\frac{19}{32}P$

Mm [@ load] . . . . = $\frac{13}{64}PL$

M_1 [@ R_2] . . . . = $-\frac{3}{32}PL$



Eq 412-40

$R_1 = V_1 \ldots = \frac{Pb}{4L^3}(4L^2 - a(L + a))$

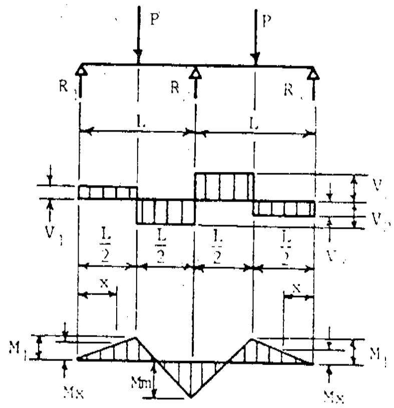
$R_2 = V_2 + V_3 \ldots = \frac{Pa}{2L^3}(2L^2 + b(L + a))$

$R_3 = V_3 \ldots = -\frac{Pab}{4L^3}(L + a)$

$V_2 \ldots = \frac{Pa}{4L^3}(4L^2 + b(L + a))$

Mm [@ load] $\ldots = \frac{Pab}{4L^3}(4L^2 - a(L + a))$

M_1 [@ R_2] $\ldots = -\frac{Pab}{4L^2}(L + a)$



Eq 412-41

$R_1 = V_1 = R_3 = V_3 \ldots = \frac{5P}{16}$

$R_2 = 2V_2 \ldots = \frac{11P}{8}$

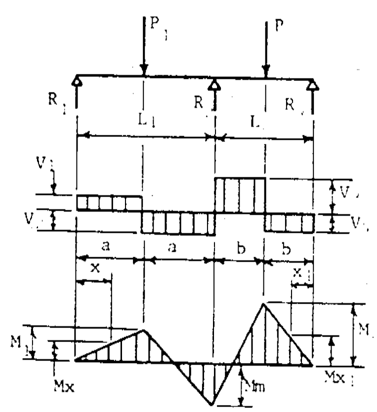
$V_2 = P - R_1 \ldots = \frac{11P}{16}$

$Mm \ldots = -\frac{3PL}{16}$

$M_1 \ldots = \frac{5PL}{32}$

Mx [@ $x < a$] $\ldots = R_1x$

Mx [@ $a < x < L$] $\ldots = R_1x - P(x - L/2)$



Eq 412-42

$R_1 = V_1 \ldots = \frac{M_3}{L_1} + \frac{P_1}{2}$

$R_2 \ldots = P_1 + P_2 - R_1 - R_3$

$R_3 = V_4 \ldots = \frac{M_3}{L_2} + \frac{P_2}{2}$

$V_2 \ldots = P_1 - R_1$

$V_3 \ldots = P_2 - R_3$

$M_1 \ldots = R_1a$

$M_2 \ldots = R_2b$

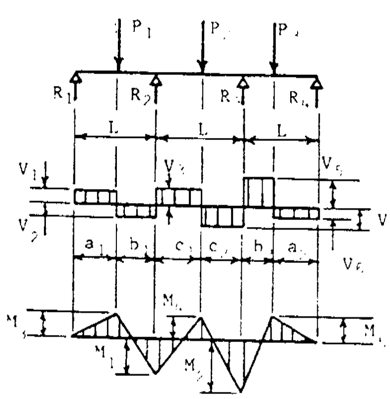
$M_3 \ldots = -\frac{3}{16}\left(\frac{P_1L_1^2 + P_2L_2^2}{L_1 + L_2}\right)$

Mx [@ $x < a$] $\ldots = R_1x$

Mx [@ $a < x < L$] $\ldots = R_1x - P(x - a)$

Mx_1 [@ $x_1 < b$] $\ldots = R_3x_1$

Mx_1 [@ $b < x_1 < L$] $\ldots = R_3x_1 - P(x_1 - b)$



Eq 412-43

$R_1 = V_1 \ldots = \frac{M_1 + P_1b_1}{L}$

$R_2 \ldots = \frac{M_2 - 2R_1L + P_2c_2 + P_1(L + b_1)}{L}$

$R_3 \ldots = \frac{M_1 - 2R_4L + P_2c_1 + P_3(L + b_2)}{L}$

$R_4 = V_6 \ldots = \frac{M_2 + P_3b_2}{L}$

$V_2 \ldots = R_1 - P_1$

$V_3 \ldots = R_2 - V_2$

$V_4 \ldots = R_3 - V_5$

$V_5 \ldots = R_4 - P_3$

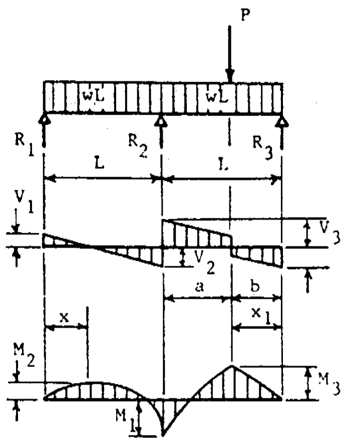
$M_1 \ldots = \frac{-4P_1a_1b_1(L + a_1) - P_2c_1c_2(7L - 5c_1) + P_3b_2a_2(L + a_2)}{15L^2}$

$M_2 \ldots = \frac{P_1a_1b_1(L + a_1) - P_2c_1c_2(2L + 5c_1) - 4P_3b_2a_2(L + a_2)}{15L^2}$

$M_3 \ldots = R_1a_1$

$M_4 \ldots = M_1 + V_3c_1$

$M_5 \ldots = R_4a_2$



Eq 412-44

$R_1 = V_1$ . . . . $= \frac{M_1}{L} - \frac{wL}{2}$

R_2 . . . . $= 2wL + P - R_1 - R_3$

$R_3 = V_4$ . . . . $= \frac{M_1 + Pa}{L} + \frac{wL}{2}$

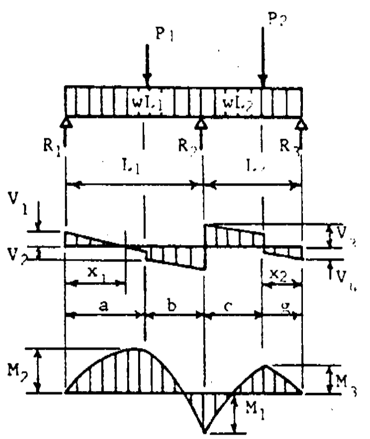
V_2 . . . . $= wL - R_1$

V_3 . . . . $= wL + P - R_3$

M_1 . . . . $= -\frac{wL^2}{8} - \frac{Pb(L^2 - b^2)}{4L^2}$

M_2 $[@\ x = \frac{R_1}{w}]$ . . . . $= R_1x - \frac{wx^2}{2}$

M_3 $[@\ x_1 = \frac{R_3}{w}$ when $R_3 < wb] = R_3x_1 - \frac{wx_1^2}{2}$



Eq 412-45

$R_1 = V_1$ . . . . $= \frac{M_1 + P_1b}{L_1} + \frac{wL_1}{2}$

R_2 . . . . $= wL_1 + wL_2 + P_1 + P_2 - R_1 - R_3$

$R_3 = V_4$ . . . . $= \frac{M_1 + P_2c}{L_2} + \frac{wL_2}{2}$

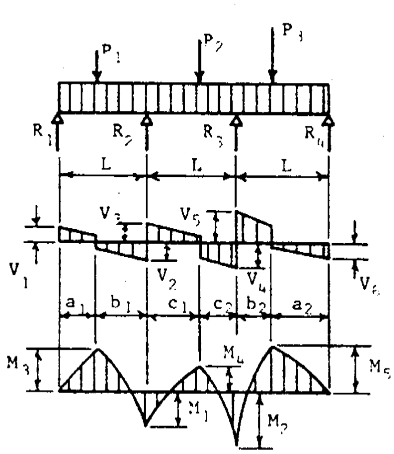
V_2 . . . . $= wL_1 + P_1 - R_1$

V_3 . . . . $= wL_2 + P_2 - R_3$

M_1 . . . . $= -\left[\frac{4P_1L_1^2(\frac{a}{L_1} - \frac{a^3}{L_1^3}) + 4P_2L_2^2(\frac{g}{L_2} - \frac{g^3}{L_2^3}) + wL_1^3 + wL_2^3}{8(L_1 + L_2)}\right]$

M_2 $[@\ \frac{R_1}{w}$ when $R_1 \leq wa]$ . . $= R_1x_1 - \frac{wx_1^2}{2}$

M_3 $[@\ \frac{R_3}{w}$ when $R_3 \leq wg]$ . . $= R_3x_2 - \frac{wx_2^2}{2}$



Eq 412-46

$R_1 = V_1$ . . . . $= \frac{wL}{2} + \frac{M_1 + P_1b_1}{L}$

R_2 . . . . $= 2wL + \frac{M_2 - 2R_1L + P_2c_2 + P_1(L + b_1)}{L}$

R_3 . . . . $= 2wL + \frac{M_1 - R_4L + P_2c_1 + P_3(L + b_2)}{L}$

$R_4 = V_6$ . . . . $= \frac{wL}{2} + \frac{M_2 + P_3b_2}{L}$

V_2 . . . . $= P_1 + wL - R_1$

V_3 . . . . $= R_2 - V_2$

V_4 . . . . $= R_3 - V_5$

V_5 . . . . $= P_3 + wL - R_4$

M_1 . . . . $= \frac{-4P_1a_1b_1(L + a_1) - P_2c_1c_2(7L - 5c_1) + P_3b_2a_2(L + a_2)}{15L^2} - \frac{wL^2}{10}$

M_2 . . . . $= \frac{P_1a_1b_1(L + a_1) - P_2c_1c_2(2L + 5c) - 4P_3b_2a_2(L + a_2)}{15L^2} - \frac{wL^2}{10}$

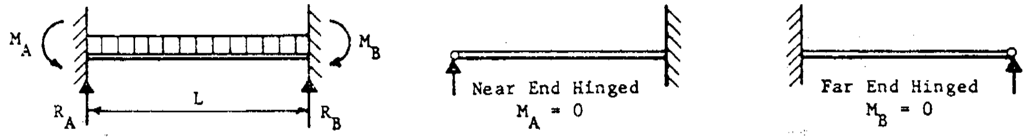
M_3 . . . . $= R_1a_1 - \frac{wa_1^2}{2}$

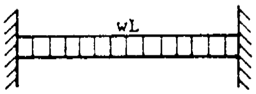
M_4 . . . . $= M_1 + V_3c_1 - \frac{wc_1^2}{2}$

M_5 . . . . $= R_4a_2 - \frac{wa_2^2}{2}$

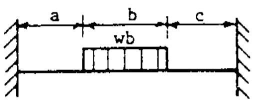
Fixed-End Beams

Eq 412-47

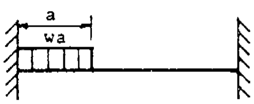




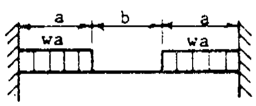
1



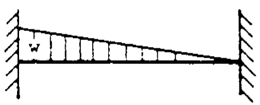
2



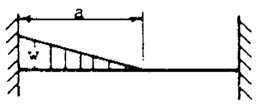
3



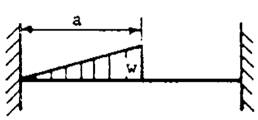
4



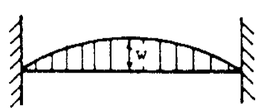
5



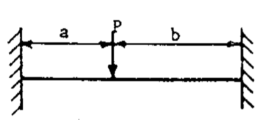
6



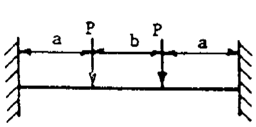
7



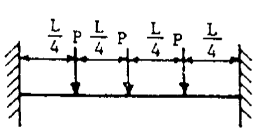
8



9



10



11

Both Ends Fixed

Fig #	M_A	M_B	R_A	R_B
1	$\frac{wL^2}{12}$	$\frac{wL^2}{12}$	$wL/2$	$wL/2$
2	$\frac{w}{12L^2}[(b+c)^3(4L-3b-3c)$ $-c^3(4L-3c)]$	$\frac{w}{12L^2}[(a+b)^3(4L-3a-3b)-a^3(4L-3a)]$	$\frac{M_A-M_B+wb(c+b/2)}{L}$	$\frac{M_B-M_A+wb(a+b/2)}{L}$
3	$\frac{wa^2}{12L^2}(3a^2-8aL+6L^2)$	$\frac{wa^3}{12L^2}(4L-3a)$	$\frac{M_A-M_B+wa(L-a/2)}{L}$	$\frac{M_B-M_A+wa^2/2}{L}$
4	$\frac{wa^2}{6L}(3L-2a)$	$\frac{wa^2}{6L}(3L-2a)$	wa	wa
5	$\frac{wL^2}{20}$	$\frac{wL^2}{30}$	$\frac{7wL}{20}$	$\frac{3wL}{20}$
6	$\frac{wa^2}{60L^2}(3a^2-10aL+10L^2)$	$\frac{wa^3}{60L^2}(5L-3a)$	$\frac{M_A-M_B+\frac{wa}{2}(L-a/3)}{L}$	$\frac{M_B-M_A+wa^2/6}{L}$
7	$\frac{wa^2}{30L^2}(10L^2-15aL+6a^2)$	$\frac{wa^3}{20L^2}(5L-4a)$	$\frac{M_A-M_B+\frac{wa}{2}(L-2a/3)}{L}$	$\frac{M_B-M_A+wa^2/3}{L}$
8	$\frac{wL^2}{15}$	$\frac{wL^2}{15}$	$wL/3$	$wL/3$
9	$\frac{Pab^2}{L^2}$	$\frac{Pa^2b}{L^2}$	$\frac{Pb+M_A-M_B}{L}$	$\frac{M_B-M_A+Pa}{L}$
10	$\frac{Pa}{L}(L-a)$	$\frac{Pa}{L}(L-a)$	P	P
11	$\frac{5PL}{16}$	$\frac{5PL}{16}$	$3P/2$	$3P/2$

Near End Hinged, $M_A = 0$

Fig #	M_A	M_B	R_A	R_B
6	0	$\frac{wa^2}{120L^2}(10L^2 - 3a^2)$	$-\frac{M_B + \frac{wa}{2}(L - a/3)}{L}$	$\frac{M_B + wa^2/6}{L}$
7	0	$\frac{wa^2}{30L^2}(5L^2 - 3a^2)$	$-\frac{M_B + \frac{wa}{2}(L - 2a/3)}{L}$	$\frac{M_B + wa^2/3}{L}$
3	0	$\frac{wa^2}{8L^2}(2L^2 - a^2)$	$-\frac{M_B + (wa)(L - a/2)}{L}$	$\frac{M_B + wa^2/2}{L}$

Far End Hinged, $M_B = 0$

Fig #	M_A	M_B	R_A	R_B
1	$wL^2/8$	0	$5wL/8$	$3wL/8$
2	$\frac{w}{8L^2}(b^2 + 2cb)(2L^2 - 2c^2 - 2cb - b^2)$	0	$\frac{M_A + wb(c + b/2)}{L}$	$-\frac{M_A + wb(a + b/2)}{L}$
3	$\frac{wa^2}{8L^2}(2L - a)^2$	0	$\frac{M_A + wa(L - a/2)}{L}$	$-\frac{M_A + wa^2/2}{L}$
4	$\frac{wa^2}{4L}(3L - 2a)$	0	$\frac{M_A + waL}{L}$	$\frac{waL - M_A}{L}$
5	$wL^2/15$	0	$2wL/5$	$wL/10$
6	$\frac{wa^2}{120L^2}(20L^2 - 15aL + 3a^2)$	0	$\frac{M_A + \frac{wa}{2}(L - a/3)}{L}$	$-\frac{M_A + wa^2/6}{L}$
7	$\frac{wa^2}{120L^2}(40L^2 - 45aL + 12a^2)$	0	$\frac{M_A + \frac{wa}{2}(L - 2a/3)}{L}$	$-\frac{M_A + wa^2/3}{L}$
8	$wL^2/10$	0	$\frac{M_A + wL^2/3}{L}$	$-\frac{M_A + wL^2/3}{L}$
9	$\frac{Pab}{L^2}(b + a/2)$	0	$\frac{Pb + M_A}{L}$	$\frac{Pa - M_A}{L}$
10	$\frac{3Pa}{2L}(L - a)$	0	$\frac{M_A + PL}{L}$	$-\frac{M_A + PL}{L}$
11	$\frac{15PL}{32}$	0	$\frac{M_A + 3PL/2}{L}$	$-\frac{M_A + 3PL/2}{L}$

Table 631-1. Insulation values.
From *1981 ASHRAE Handbook of Fundamentals*. Values do not include surface conditions unless noted otherwise. All values are approximate.

Material	R-value Per inch (approximate) 1/k	R-value For thickness listed 1/C
Batt and blanket insulation		
Glass or mineral wool, fiberglass	3.00-3.80*	
Fill-type insulation		
Cellulose	3.13-3.70	
Glass or mineral wool	2.50-3.00	
Vermiculite	2.20	
Shavings or sawdust	2.22	
Hay or straw, 20″		30+
Rigid insulation		
Exp. polystyrene,		
extruded, plain	5.00	
molded beads, 1 pcf	5.00	
molded beads, over 1 pcf	4.20	
Expanded rubber	4.55	
Expanded polyurethane, aged	6.25	
Glass fiber	4.00	
Wood or cane fiberboard	2.50	
Polyisocyanurate	7.04	
Foamed-in-place insulation		
Polyurethane	6.00	
Building materials		
Concrete, solid	0.08	
Concrete block, 3 hole, 8″		1.11
lightweight aggregate, 8″		2.00
lightweight, cores insulated		5.03
Brick, common	0.20	
Metal siding	0.00	
hollow-backed		0.61
insulated-backed, 3/8″		1.82
Softwoods, fir and pine	1.25	
Hardwoods, maple and oak	0.91	
Plywood, 3/8″	1.25	0.47
Plywood, 1/2″	1.25	0.62
Particleboard, medium density	1.06	
Hardboard, tempered, 1/4″	1.00	0.25
Insulating sheathing, 25/32″		2.06
Gypsum or plasterboard, 1/2″		0.45
Wood siding, lapped, 1/2″x8″		0.81
Asphalt shingles		0.44
Wood shingles		0.94
Windows (includes surface conditions)		
Single glazed		0.91
with storm windows		2.00
Insulating glass, 1/4″ air space		
double pane		1.69
triple pane		2.56
Doors (exterior, includes surface conditions)		
Wood, solid core, 1 3/4″		3.03
Metal, urethane core, 1 3/4″		2.50
Metal, polystyrene core, 1 3/4″		2.13
Air space (3/4″ to 4″)		0.90
Surface conditions		
Inside surface		0.68
Outside surface		0.17

*The insulation value of fiberglass varies with batt thickness. Check package label.

Table 631-3. Insulation R-values for other construction.

	R_T
Roofs	
Metal roofing, 25⁄32″ insulating sheathing	2.91
Metal roofing, 0.4″ expanded polyurethane	3.35
Metal roofing, 1″ molded polystyrene, 1 pcf	5.85
Ceilings	
2″ expanded polyurethane	13.35
½″ plywood, 4″ glass or mineral wool fill insulation	13.47
Metal roofing, R-19 blanket insulation	19.85
½″ plywood, 8″ glass or mineral wool fill insulation	25.47
Doors	
½″ plywood, R-2 blanket insulation, ¾″ air space, ½″ plywood	4.99
½″ plywood, 1″ polystyrene molded, 1 pcf ¾″ air space, ½″ plywood	7.99
Floor perimeter (per foot of exterior wall)	
Concrete	1.23
Concrete, with 2″x24″ of rigid insulation around perimeter	2.22