<table>
<thead>
<tr>
<th>Topic</th>
<th>Label</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumber dimensions</td>
<td>Table 201-6</td>
<td>201.5</td>
</tr>
<tr>
<td>Nail sizes</td>
<td>Table 205-2</td>
<td>205.2</td>
</tr>
<tr>
<td>^1Portland Cement types</td>
<td>Left column</td>
<td>207.1</td>
</tr>
<tr>
<td>Lumber section properties</td>
<td>Table 405-3</td>
<td>405.2</td>
</tr>
<tr>
<td>^1Equivalent fluid density</td>
<td></td>
<td>410.1</td>
</tr>
<tr>
<td>^1Janssen’s equation</td>
<td></td>
<td>411.1-2</td>
</tr>
<tr>
<td>Beam Formulas</td>
<td></td>
<td>412.2-412.15</td>
</tr>
<tr>
<td>Insulation values</td>
<td>Table 631-1</td>
<td>631.4</td>
</tr>
<tr>
<td></td>
<td>Table 631-3</td>
<td>631.6</td>
</tr>
</tbody>
</table>
Commercial Lumber

Most commercial lumber in farm buildings is softwood lumber graded under the American Softwood Lumber Standard (ASLS). This standard is a common basis for uniform, industry-wide inspection and grade marking practices for each piece of lumber produced and sold under its provisions. It also provides for coordinating the grades of species and preparing grading rules for each species. Lumber meeting the minimum size and grade provisions of the ASLS is graded under the rules of the various regional associations that have been approved by the Board of Review of the ASLS Committee.

Lumber is produced for various end uses, in a variety of sizes and moisture contents, and in several manufacturing classifications. Lumber is classified according to end use:

Yard lumber is for ordinary construction and general building purposes and is commonly available in local lumber yards.

Structural lumber is for framing requiring specific strength. Some structural lumber is also available from local lumber yards.

Factory and shop lumber is intended for remanufacturing.

Boards are less than 2" in nominal thickness and 2" or more in nominal width.

Dimension lumber is from 2" to less than 5" in nominal thickness and 2" or more in nominal width.

Timbers are 5" nominal or more in least dimension.

Lumber is classified according to nominal size, Table 201-6. It is also classified according to extent of manufacture:

Rough lumber has not been dressed (surfaced) but has been sawed, edged, and trimmed at least to saw marks showing in the wood on the four longitudinal surfaces of each piece for its overall length.

Dressed (surfaced) lumber has been planed to smooth surfaces and uniform size on one side (S1S), two sides (S2S), one edge (S1E), two edges (S2E), or a combination of sides and edges (S1S1E, S1S2E, S2S1E, S4S).

Worked lumber has been dressed, matched, shiplapped, or patterned.

Matched lumber has been worked to a tongue on one edge of each piece and a groove on the opposite edge for close tongue-and-groove joints by fitting two pieces together. When end-matched, the tongue and groove are also worked in the ends.

Shiplapped lumber has been worked or rabbed on both edges of each piece for a close lapped joint between two pieces.

Patterned lumber is further worked to a pattern or molded form.

Lumber is surfaced either dry or green. Dry lumber has been seasoned or dried to a moisture content of 19% or less. Green lumber has a % mc over 19%. Lumber is surfaced green at a larger size than when surfaced dry so it is at the same size in service whether it was surfaced dry or green.

Table 201-6. Nominal and minimum dressed lumber sizes.

<table>
<thead>
<tr>
<th>Item</th>
<th>Thicknesses</th>
<th>Face widths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nominal</td>
<td>Dressed</td>
</tr>
<tr>
<td>Boards</td>
<td>1</td>
<td>¾ in.</td>
</tr>
<tr>
<td></td>
<td>1¼</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>1½</td>
<td>1½</td>
</tr>
<tr>
<td></td>
<td>½</td>
<td>2¾</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3½</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4½</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5½</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6½</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7½</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8½</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9½°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11½°</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13½°</td>
</tr>
<tr>
<td>Dimension</td>
<td>2</td>
<td>1½</td>
</tr>
<tr>
<td></td>
<td>2½</td>
<td>2½</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3½</td>
</tr>
<tr>
<td></td>
<td>3½</td>
<td>4½</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>4½</td>
</tr>
<tr>
<td></td>
<td>4½</td>
<td>5½</td>
</tr>
<tr>
<td></td>
<td>5 and</td>
<td>½ off</td>
</tr>
<tr>
<td>Timbers (Dressed</td>
<td>thicker</td>
<td>wider</td>
</tr>
</tbody>
</table>

There are three regional associations that grade most softwood lumber: the Southern Pine Inspection Bureau (SPIB), the West Coast Lumber Inspection Bureau (WCLB), and the Western Wood Products Association (WWPA). Even though the grading rules of these three associations conform to the American Lumber Standard, there are variations in the names assigned to comparable grades.

Typical lumber grades and suggested uses for farm construction are in Table 201-7. For economy, select the lowest grade that is satisfactory for the intended use.

Most yard and structural lumber is grade stamped to help the consumer identify a given piece’s grade. In addition to the grade, the stamp normally indicates the grading association, the mill that manufactured the lumber, the species, and the moisture content at which the lumber was surfaced. Typical grade stamps of two different grading associations are shown in Fig 201-7.

Designing With Lumber


Because strength is a function of moisture content, and member selection is based on both the size of the member and its strength, three elements are established before designing in wood: moisture content during use, size during use, and quality or allowable stresses.
The allowable withdrawal load per nail in toenailed joints for all conditions of seasoning is ⅔ of calculated withdrawal resistance. Lead holes prevent or reduce splitting and increase withdrawal resistance. Clinching increases withdrawal strength up to 170%. Clinch nails across the grain.

Small nails in plywood cause less splitting when near an edge. Withdrawal resistance is 70%-85% that of solid wood. For plywood less than ½" thick, less splitting offsets the lower withdrawal resistance. The withdrawal load per inch decreases with increase in the number of plys. Face grain direction has little influence.

**Lateral Resistance**

The allowable load for a bright, common wire nail in lateral resistance when driven into the side grain (perpendicular to the wood fibers) of seasoned wood is:

$$ P = K D^{0.2} $$

- **P** = the allowable lateral load, lb/nail
- **K** = a factor allowing for wood species and for safety factor
- **D** = the diameter of the nail, inches

Ultimate lateral nail loads in softwoods are about six times, and in hardwoods about 11 times, the load P. The side member and the member holding the nail point have about the same density, and nail penetration is not less than 10D for dense woods and 14D for light woods. When metal is held to wood, increase the allowable lateral nail load about 25%.

For end nailing, reduce P to about 60%.

**Spacing**

The NDS gives no guide on nail spacing. The following are from FPL report 0100, 1965:

- Nail spacing, minimum:
  - 1½" edge distance
  - 1" across grain
  - 2½" parallel to grain
  - 2¼" end distance in tension members

**Spikes**

Common wire spikes are manufactured the same as common wire nails. They have either a chisel point or a diamond point and are made in lengths of 3"-12". For corresponding lengths (3"-6"), they have larger diameters than the common wire nails, and beyond 60d they are usually designated by inches of length.

The allowable withdrawal and lateral resistance formulas and limitations given for common wire nails are applicable to spikes, except that in calculating the withdrawal load for spikes, the depth of penetration should be reduced by ⅔ the length of the point.

---

**Fastener Selection**

Much of this material is abstracted from the NDS and applies primarily to the more common structural woods. Refer to NDS for details for other species.

**Table 205-1. Specific gravity, G, of woods.**

Based on oven-dry weight and volume.

<table>
<thead>
<tr>
<th>Group</th>
<th>Typical species</th>
<th>Specific gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Birch, Maple</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>Ash</td>
<td>0.62</td>
</tr>
<tr>
<td>II</td>
<td>Douglas fir-larch</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>southern pine</td>
<td>0.55</td>
</tr>
<tr>
<td>III</td>
<td>California Redwood; Idaho white,</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>Ponderosa, sugar, and red pines</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Douglas fir, south; southern cypress</td>
<td>0.48</td>
</tr>
<tr>
<td>IV</td>
<td>Balsam fir, E. white pine</td>
<td>0.38</td>
</tr>
</tbody>
</table>

**Nails and Spikes**

Allowable loads are per nail and may be adjusted for duration of loading. For lumber pressure-treated with fire retardants, reduce nail and spike loads to:

- 90% in withdrawal if kiln dried after treatment.
- 22¾% in withdrawal if not kiln dried after treatment.
- 66¾% in lateral resistance.

**Table 205-2. Nail sizes.**

"Steel nails" are threaded, hardened steel.

<table>
<thead>
<tr>
<th>Penny-weight</th>
<th>Length in.</th>
<th>Wire diameter, in. Wire nails</th>
<th>Steel nails</th>
<th>Spikes</th>
</tr>
</thead>
<tbody>
<tr>
<td>4d</td>
<td>1½&quot;</td>
<td>0.098&quot;</td>
<td>0.120&quot;</td>
<td></td>
</tr>
<tr>
<td>6d</td>
<td>2</td>
<td>0.113</td>
<td>0.120</td>
<td></td>
</tr>
<tr>
<td>8d</td>
<td>2½</td>
<td>0.131</td>
<td>0.120</td>
<td></td>
</tr>
<tr>
<td>10d</td>
<td>3</td>
<td>0.148</td>
<td>0.135</td>
<td>0.192&quot;</td>
</tr>
<tr>
<td>12d</td>
<td>3¼</td>
<td>0.148</td>
<td>0.135</td>
<td>0.192</td>
</tr>
<tr>
<td>16d</td>
<td>3½</td>
<td>0.162</td>
<td>0.148</td>
<td>0.207</td>
</tr>
<tr>
<td>20d</td>
<td>4&quot;</td>
<td>0.192&quot;</td>
<td>0.177&quot;</td>
<td>0.225&quot;</td>
</tr>
<tr>
<td>30d</td>
<td>4½&quot;</td>
<td>0.207</td>
<td>0.177</td>
<td>0.244</td>
</tr>
<tr>
<td>40d</td>
<td>5</td>
<td>0.225</td>
<td>0.177</td>
<td>0.263</td>
</tr>
<tr>
<td>50d</td>
<td>5½&quot;</td>
<td>0.244</td>
<td>0.177</td>
<td>0.283</td>
</tr>
<tr>
<td>60d</td>
<td>6</td>
<td>0.262</td>
<td>0.177</td>
<td>0.283</td>
</tr>
<tr>
<td>70d</td>
<td>7&quot;</td>
<td></td>
<td>0.207</td>
<td>0.312</td>
</tr>
<tr>
<td>80d</td>
<td>8</td>
<td></td>
<td>0.207</td>
<td>0.357</td>
</tr>
<tr>
<td>90d</td>
<td>9</td>
<td></td>
<td>0.207</td>
<td></td>
</tr>
<tr>
<td>⅜d</td>
<td>7</td>
<td></td>
<td>0.312</td>
<td></td>
</tr>
<tr>
<td>⅜d</td>
<td>8½&quot;</td>
<td></td>
<td>0.375</td>
<td></td>
</tr>
</tbody>
</table>

**Table 205-3. Wire nails per pound.**

<table>
<thead>
<tr>
<th>Name</th>
<th>Nails/lb</th>
<th>Name</th>
<th>Nails/lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>8d common</td>
<td>106</td>
<td>8d floor brads</td>
<td>99</td>
</tr>
<tr>
<td>20d &quot;</td>
<td>31</td>
<td>8d casing</td>
<td>145</td>
</tr>
<tr>
<td>60d &quot;</td>
<td>11</td>
<td>3d brads</td>
<td>568</td>
</tr>
<tr>
<td>6d finish</td>
<td>309</td>
<td>3d fine</td>
<td>778</td>
</tr>
<tr>
<td>8d &quot;</td>
<td>189</td>
<td>3d shingle</td>
<td>429</td>
</tr>
<tr>
<td>10d &quot;</td>
<td>121</td>
<td>4d &quot;</td>
<td>274</td>
</tr>
</tbody>
</table>
207 CONCRETE AS A MATERIAL

Concrete is a mixture of Portland cement, water, air and aggregates. Portland cement is sold in bulk, or in bags of one cubic foot (94 lb). The aggregates provide volume at low cost, composing 66%-78% of the concrete.

Cement and water form a paste that hardens and glues the aggregates together. The quality of concrete is directly related to the binding qualities of this cement paste.

Selection

General properties

Concrete is very durable and resists attack by water, animal manures, chemicals such as fertilizers, and fire. Use high quality concrete around milk, silage, and animal manure.

Concrete is very weak in tension. Its strength in compression depends on the proportions of the mix. The compressive strength is 2 to 5 times that of wood. Most structural uses involve reinforced concrete, which depends on concrete's strength in compression and steel's strength in tension.

Concrete can be finished in either smooth or rough texture. It can be colored with pigments or painted.

Portland cement

Portland cement got its name from the Isle of Portland, near England, where it was first developed. This improvement over an ancient building material is made by burning limestone and clay, adding gypsum, and grinding the result to a fine consistent powder.

A number of variations have been developed for special purposes:

- Type I, normal Portland cement, is the general purpose type, the most common, and is usually furnished unless an alternative is specified.
- Type II is modified to release less heat during curing, and is therefore suitable in mass concrete—heavy retaining walls, or deadmen for suspension bridges. It is moderately high in resistance to sulphates. Type II is replacing Type I as the basic type in some areas.
- Type III is high-early-strength; it is very finely ground and sets very rapidly. It is useful for slipform construction and for cold weather jobs.
- Type IV is the lowest heat variety, and while suitable for large masses, develops strength relatively slowly. Its primary use is for mass concrete dams and other large volume structures.
- Type V is especially sulphate-resistant and cures more slowly than Type I.
- Types I, II, and III are available as Types IA, IIA, and IIIA. These are air-entrained Portland cements formulated with a compound that releases many tiny bubbles of air during curing. The resulting concrete is highly resistant to frost action and has some increased resistance to salts. Their use is recommended for all outdoor paving and for concrete exposed to animal wastes, even though slightly weaker than Type I, II, and III.

Air-entrained concrete

An air-entraining agent added to the cement produces millions of tiny bubbles in the concrete giving the concrete greater weathering resistance. Air entraining also reduces the strength and increases the concrete workability. Because the air-entraining agent increases the weathering resistance compared to its reduction of strength, air entraining is usually recommended for all concrete subjected to freezing and thawing. Table 207-1 relates the amounts of air entraining to aggregate size.

Other additives include pigments for coloring, gypsum to retard setting time, calcium to reduce setting time, and bentonite to improve workability.

Table 207-1. Air content for air-entrained concrete.

<table>
<thead>
<tr>
<th>Max. aggregate size</th>
<th>Amount of air, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1½&quot;, 2&quot;, or 2½&quot;</td>
<td>4 to 6</td>
</tr>
<tr>
<td>¾&quot; or 1&quot;</td>
<td>5 to 7</td>
</tr>
<tr>
<td>½&quot; or ¾&quot;</td>
<td>6½ to 8½</td>
</tr>
</tbody>
</table>

Water

Cement in concrete cures by the chemical combination of water and cement—not by drying like sheets in the sun. Use water for concrete that is essentially good enough to drink, without harmful chemicals, trash, or organic matter.

The strength of concrete is greatly dependent on the water-cement ratio. Enough water is needed for full curing, but excess water leaves voids when it evaporates. If a mix is too stiff to handle well, do not add water; reject the batch or add cement and water—reduce the amount of aggregates in subsequent batches.

Aggregates

Sand and gravel, the usual aggregates, are glued together by the cured cement paste. They must be free of, or very low in, silt and organic matter, and they must be hard and strong.

Particles up to ¾" are sand, and above ¾" are gravel. Well graded aggregates occupy most of the volume of the concrete, minimizing the amount of cement paste, which is the expensive ingredient. Using spheres for illustration, large balls have less surface area than small ones, requiring less cement. But small balls fit in between the large ones to help fill up the voids.
Table 405-2. 1982 allowable stresses in tension parallel to grain, lb/in².
Lumber 2" to 4" thick, wider than 4", and used at 19% m.c. max.

<table>
<thead>
<tr>
<th>Use</th>
<th>Grade</th>
<th>Lumber width</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5&quot; , 6&quot;</td>
</tr>
<tr>
<td>Douglas Fir (WCLIB)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Joist &amp; plank</td>
<td>Select</td>
<td>1200</td>
</tr>
<tr>
<td>No. 1</td>
<td>1000</td>
<td>800</td>
</tr>
<tr>
<td>No. 2</td>
<td>650</td>
<td>520</td>
</tr>
<tr>
<td>Stud</td>
<td>375</td>
<td></td>
</tr>
<tr>
<td>Southern Pine (SPIB)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Struct</td>
<td>Select</td>
<td>1150</td>
</tr>
<tr>
<td>Joist &amp; plank</td>
<td>No. 1</td>
<td>975</td>
</tr>
<tr>
<td>No. 2</td>
<td>625</td>
<td>500</td>
</tr>
<tr>
<td>Stud</td>
<td>350</td>
<td></td>
</tr>
</tbody>
</table>

Fig 405-1. Equilibrium moisture content of wood.
Wood Handbook, No. 72, USDA.
Even in livestock buildings, it is unlikely that relative humidity is high enough long enough to raise the moisture content of lumber appreciably above 15%. Lumber shrinks during drying, so NDS stresses apply to moisture content and size at time of surfacing. NDS stresses still apply if the lumber is further dried, even though it shrinks.

Table 405-3. Lumber section properties.

<table>
<thead>
<tr>
<th>Nominal Bx0, in.</th>
<th>Dressed Bx0, in.</th>
<th>Area A, in²</th>
<th>Inertia moment I, in⁴</th>
<th>Section modulus S, in³</th>
<th>Shear factor S, in²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 2</td>
<td>3/4x1 1/2</td>
<td>1.13</td>
<td>0.21</td>
<td>0.28</td>
<td>0.75</td>
</tr>
<tr>
<td>x 3</td>
<td>x2 1/2</td>
<td>1.88</td>
<td>0.98</td>
<td>0.78</td>
<td>1.25</td>
</tr>
<tr>
<td>x 4</td>
<td>x3 1/2</td>
<td>2.63</td>
<td>2.68</td>
<td>1.53</td>
<td>1.75</td>
</tr>
<tr>
<td>x 5</td>
<td>x4 1/2</td>
<td>3.38</td>
<td>5.70</td>
<td>2.53</td>
<td>2.25</td>
</tr>
<tr>
<td>x 6</td>
<td>x5 1/2</td>
<td>4.13</td>
<td>10.40</td>
<td>3.78</td>
<td>2.75</td>
</tr>
<tr>
<td>x 7</td>
<td>x6 1/2</td>
<td>4.88</td>
<td>17.16</td>
<td>5.28</td>
<td>3.25</td>
</tr>
<tr>
<td>1 x 8</td>
<td>3/4x7 1/4</td>
<td>5.44</td>
<td>23.82</td>
<td>6.57</td>
<td>3.63</td>
</tr>
<tr>
<td>x 9</td>
<td>x8 1/4</td>
<td>6.19</td>
<td>35.09</td>
<td>8.51</td>
<td>4.13</td>
</tr>
<tr>
<td>x 10</td>
<td>x9 1/4</td>
<td>6.94</td>
<td>49.47</td>
<td>10.70</td>
<td>4.63</td>
</tr>
<tr>
<td>x 11</td>
<td>x10 1/4</td>
<td>7.69</td>
<td>67.31</td>
<td>13.13</td>
<td>5.13</td>
</tr>
<tr>
<td>x 12</td>
<td>x11 1/4</td>
<td>8.44</td>
<td>88.99</td>
<td>15.82</td>
<td>5.63</td>
</tr>
<tr>
<td>2 x 2</td>
<td>1 1/2x1 1/2</td>
<td>2.25</td>
<td>0.42</td>
<td>0.56</td>
<td>1.50</td>
</tr>
<tr>
<td>x 3</td>
<td>x2 1/2</td>
<td>2.65</td>
<td>0.95</td>
<td>1.56</td>
<td>2.50</td>
</tr>
<tr>
<td>x 4</td>
<td>x3 1/2</td>
<td>3.25</td>
<td>3.35</td>
<td>3.06</td>
<td>3.50</td>
</tr>
<tr>
<td>x 5</td>
<td>x4 1/2</td>
<td>3.85</td>
<td>5.15</td>
<td>5.06</td>
<td>4.50</td>
</tr>
<tr>
<td>x 6</td>
<td>x5 1/2</td>
<td>4.45</td>
<td>7.95</td>
<td>7.56</td>
<td>5.50</td>
</tr>
<tr>
<td>x 8</td>
<td>x7 1/4</td>
<td>10.88</td>
<td>47.63</td>
<td>13.14</td>
<td>7.25</td>
</tr>
<tr>
<td>2 x 10</td>
<td>1 1/2x9 1/4</td>
<td>13.88</td>
<td>98.93</td>
<td>21.39</td>
<td>9.25</td>
</tr>
<tr>
<td>x 12</td>
<td>x11 1/4</td>
<td>16.88</td>
<td>177.98</td>
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*Full 12" wide.*
Rankine's theory comes from classical soils mechanics. It relates lateral to vertical pressure in non-cohesive soils, such as sand and gravel, which are similar in behavior to small grains. It yields adequate equivalent fluid density values for designing shallow grain bin walls. See Chapter 103 for discussion of shallow vs. deep bins.

A companion theory, Coulomb's, analyzes forces on a sliding wedge of material behind a wall. It yields the same values as Rankine's for the angle of the failure plane and the ratio of vertical to lateral pressures when wall friction is neglected.

If a wall restraining the grain moves slightly away from the grain, the lateral pressure decreases, the shear stress increases, and the vertical pressure at the bottom of the grain mass remains unchanged. A plane of shear failure forms and the wedge of grain between it and the wall flows toward the wall as it moves. The failure plane that produces maximum horizontal forces when the pile is level and the wall is vertical is $45^\circ + \phi/2$ from the horizontal, Fig 410-1.

![Fig 410-1. Failure plane and emptying angle of repose in grain.](image1)

Rankine's definition of the ratio of lateral to vertical pressure for the above condition is:

$$k = \tan^2(45 - \phi/2) = (1 - \sin \phi)/(1 + \sin \phi)$$

Eq 410-1

The equivalent fluid density of a grain is:

$$EFD = (\text{unit weight}) \times k = w \times k$$

Using corn as an example:

- Its emptying angle of repose, $\phi = 27^\circ$, is the angle of internal friction.
- Therefore:
  
  $$k = (1 - \sin 27^\circ)/(1 + \sin 27^\circ) = 0.38$$
  
  $$EFD = (48 \text{ lb/ft}^2) \times 0.38 = 18 \text{ lb/ft}^2$$
  
  The resulting load diagram on a wall is in Fig 410-2.

The ease of an inclined grain surface is more complicated. Lateral pressure is assumed to act at an angle equal to the angle of the coefficient of friction between the wall and the grain mass. If this friction is neglected, which it usually is, then the direction of the thrust is horizontal.

![Fig 410-2. Load on a wall from a flat grain surface.](image2)

Use the more generalized equation for $k$ if the grain surface or wall is inclined, Fig 410-3.

$$k = \sin^2(\theta-\phi) + \sin^2(\theta) \times \sin(\theta + \beta) \times T^2$$

$$T = 1 + [\sin(\phi + \beta) \times \sin(\phi - \alpha) + \sin(\theta - \alpha) \times \sin(\theta + \beta)]^{1/2}$$

Eq 410-3

- $k$ = ratio of lateral to vertical internal pressure
- $\phi$ = angle of internal friction, usually taken as the emptying angle of repose, degrees
- $\theta$ = angle of wall from horizontal, degrees
- $\beta$ = angle of wall friction, degrees
- $\alpha$ = filling angle of repose, degrees

Example, for corn:

- $\phi = 27^\circ$
- $\theta = 90^\circ$
- $\beta = 0^\circ$
- $\alpha = 16^\circ$
- $T = [(\sin 27-0)(\sin 27-16) + \sin(90-16)\sin(90+0)]^{1/2} = 1.3$

- $k = \sin^2(90-27)\sin^2(90)\sin(90) \times 1.3^2 = 0.4696$

- $EFD = (48 \text{ lb/ft}^2) \times (0.4696) = 22.5 \text{ lb/ft}^2$
Many grain bins have significant wall friction with resulting lower vertical and lateral pressures than predicted by equivalent fluid density. The distinction between deep and shallow bins (those with and without significant wall friction) is discussed in Chapter 103.

The interaction of pressures on opposite bin walls has the effect of creating a 'pressure dome,' Fig 411-1b, which transmits pressure laterally to the sidewalls, creating reactions comparable to those in a structural arch. The lateral bin pressure is accompanied by vertical pressures in the bin walls. The static vertical component is equal to the static lateral pressure times the coefficient of friction between the grain and the bin wall.

Janssen’s equation predicts static lateral and static vertical pressures within a grain mass. Because pressures during dynamic or unloading conditions are often greater than static pressures, do not use Janssen’s equation directly for bin design. Over-pressure factors and other modifications of these static pressures are discussed in Chapter 103.

A detailed derivation of Janssen’s equation is illustrated in the following material.

Nomenclature:
- \( \phi \) = angle of internal friction of the grain (emptying angle of repose)
- \( \beta \) = angle of friction of the grain on the bin walls
- \( \mu' = \tan \beta \) = coefficient of friction of grain on the bin walls
- \( w = \text{weight of grain, lb/ft}^2 \)
- \( F = \text{vertical pressure, lb/ft}^2 \)
- \( L_s = \text{static lateral pressure, lb/ft}^2 \)
- \( A = \text{bin area, ft}^2 \)
- \( U = \text{bin circumference, ft} \)
- \( R = A/U = \text{hydraulic radius of the bin, ft} \)
- \( H_s = \text{height of the grain, ft} \)
- \( D = \text{bin diameter, ft} \)

The bin in Fig 411-1a has a uniform area, A, a constant circumference, U, and is filled with grain weighing \( w \text{ lb/ft}^2 \) having an emptying angle of repose, \( \phi \). Let \( F \) be the vertical pressure and \( L \) the lateral pressure at any point, with both \( F \) and \( L \) assumed constant on any horizontal plane.

The weight of the grain between the sections of \( y \) and \( y + dy \) is \( (Aw)dy \); the total frictional force acting upwards at the circumference is \( (LU)(\tan \beta)dy \); the total perpendicular pressure on the upper surface is \( VA \); and total pressure on the lower surface is \( (F + dF)A \).
The static vertical pressures must be in equilibrium with their sum equal to zero.
\[ 0 = FA - (F + dF)A + (Aw)dy - (LU)(\tan \beta)dy \]
Simplify:
\[ dF = (w - L(\tan \beta)U/A)dy \]

In a granular mass, the static lateral pressure at any point is equal to the static vertical pressure at that point times a constant for the particular grain, \( k \).
\[ L = kV \]
The value of \( k \) may be determined by experiment, but the value generally used is the one derived by the Rankine method:
\[ k = \frac{(1 - \sin \beta)}{(1 + \sin \beta)} \]
It is an approximation that ignores the friction between the grain and the bin walls.
Also let \( R = A/U \) (the hydraulic radius) and \( \mu' = \tan \beta \).

Substitute:
\[ dF = (w - kF\mu'/R)dy \]
Let:
\[ k\mu'/R = n \]
Substitute again:
\[ dF = (w - nF)dy, \text{ or:} \]
\[ dF/(w - nF) = dy \]
Multiply both sides of the equation by \(-n\) and integrate:
\[ \ln(w-nF) = -ny + C \]
Evaluate the constant \( C \) at \( y = 0 \) where \( F = 0 \):
\[ C = \ln(w) \]
Rearrange:
\[ \ln(w - nF/w) = -ny \]
Take the exponential of each side of the equation:
\[ (w - nF)/w = e^{-ny} \]
Solve for \( F \):
\[ F = w(1 - e^{-ny})/n \]
Substitute for \( n \). The equation becomes:
\[ F = (wR/k\mu')(1 - e^{-yR}) \]
**Simple Beams**

**Eq 412-1**

\[ R = V \]  
\[ V_x = \frac{w(L_2 - x)}{2} \]  
\[ M_m [a \leq L/2] = \frac{w(L_2 - x)}{2} \]  
\[ M_x = \frac{w(L - x)}{2} \]  
\[ D_m [a \leq L/2] = \frac{5wl^4}{384EI} \]  
\[ D_x = \frac{wx^3}{24EI} \left( L^4 - 2Lx^2 + x^4 \right) \]

**Eq 412-2**

\[ R_1 = R_2 = V_1 = V_2 = \frac{wb}{2} \]  
\[ V_x [a < x < (a + b)] = \frac{wb}{2} - w(x - a) \]  
\[ M_m [a \leq L/2] = \frac{wb}{8} (2L - b) \]  
\[ M_x [a < x < a] = \frac{wb}{2} \]  
\[ M_x [a < x < (a + b)] = \frac{wbx}{2} - \frac{w(x - a)^2}{2} \]  
\[ D_m [a \leq L/2] = \frac{5wl^4}{384EI} - \frac{wa^2}{48EI} \left( 3L^2 - 2a^2 \right) \]

**Eq 412-3**

\[ R_1 = V_1 \text{ max. } a < c \]  
\[ R_2 = V_2 \text{ max. } a > c \]  
\[ V_x [a < x < (a + b)] = \frac{wb}{2L} (2c + b) \]  
\[ V_m [a < x < (a + b)] = \frac{wb}{2L} (2a + b) \]  
\[ M_x [a < x < a] = \frac{wb}{2} \]  
\[ M_x [a < x < (a + b)] = \frac{wbx}{2} - \frac{w(x - a)^2}{2} \]  
\[ D_x [a < x < L] = R_2 (L - x) \]  
\[ D_x \text{ when } a \text{ and } c < L/2 = \frac{5wl^4}{384EI} - \frac{w}{96EI} \left( 3L^2a^2 - 2a^4 \right) \]

**Eq 412-4**

\[ R_1 = V_1 \text{ max. } a \]  
\[ R_2 = V_2 \]  
\[ V_x [a < x] = R_1 - w(x - a) \]  
\[ M_m [a < x] = \frac{R_2}{2} \]  
\[ M_x [a < x] = \frac{R_1 - w(x - a)}{2} \]  
\[ D_x [a < x < L] = \frac{R_1}{24EI} \left( a^2 (2L - a)^2 - 2ax^2 (2L - a) + x^4 \right) \]  
\[ D_x [a < x < L] = \frac{R_1}{24EI} \left( 4axL - 2x^2 - a^2 \right) \]
Eq 412-5
\[ R_1 = V_1 = \frac{w_1a(2L - a) + w_2c^2}{2L} \]
\[ R_2 = V_2 = \frac{w_2c(2L - c) + w_1a^2}{2L} \]

\[ V_\theta = \begin{cases} w_1a > w_2c & \Rightarrow R_1 \\ w_1a < w_2c & \Rightarrow R_2 \end{cases} \]

\[ V_\chi = \begin{cases} w_1a > w_2c & \Rightarrow R_1 = w_1a \\ w_1a < w_2c & \Rightarrow R_2 \end{cases} \]

\[ M_\theta = \begin{cases} w_1a & \Rightarrow R_1 < w_1a \\ \frac{R_1^2}{2w_1} & \Rightarrow R_1 \end{cases} \]

\[ M_\chi = \begin{cases} w_1a & \Rightarrow R_1 \end{cases} \]

\[ M_x = \begin{cases} 0 < x < a & \Rightarrow R_1 - \frac{w_1a}{2} \\ R_1 - \frac{w_1a}{2} < x < (a + b) & \Rightarrow R_1 - \frac{w_1a}{2}(2x - a) \end{cases} \]

\[ D_x = \begin{cases} \text{when } a \text{ and } c < L/2 & \Rightarrow \frac{w_1a^2}{96EI}(3L^2 - 2a^2) + \frac{w_2c^2}{96EI}(3L^2 - 2c^2) \end{cases} \]

Eq 412-6
\[ W = \frac{wL}{2} \]
\[ R_1 = V_1 = \frac{wL}{6} \]
\[ R_2 = V_{\text{max}} = \frac{wL}{3} \]

\[ V_\chi = \frac{wL}{6} - \frac{w_2c^2}{2L} \]

\[ M_\chi = \frac{wL}{6} = \frac{w_2c^2}{2L} = 0.0642wL^2 \]

\[ M_\chi = \frac{w_2c^2}{6L} 

\[ D_\chi = \begin{cases} \text{when } a < L/2 & \Rightarrow \frac{w_2c^2}{360EI} (3a^4 - 10a^2x^2 + 7L^4) \end{cases} \]

Eq 412-7
\[ W = \frac{wL}{2} \]
\[ R = V = \frac{wL}{4} \]

\[ V_\chi = \begin{cases} \text{when } x < \frac{L}{2} & \Rightarrow \frac{w}{4L}(L^2 - 4x^2) \end{cases} \]

\[ M_\chi = \begin{cases} \text{when } Center & \Rightarrow \frac{wL}{12} \end{cases} \]

\[ M_\chi = \begin{cases} \text{when } x < \frac{L}{2} & \Rightarrow \frac{wL}{2}(\frac{L}{2} - 2x^2) \end{cases} \]

\[ D_\chi = \begin{cases} \text{when } Center & \Rightarrow \frac{wL}{120EI} \end{cases} \]

\[ D_\chi = \begin{cases} \text{when } x < \frac{L}{2} & \Rightarrow \frac{wL}{960EI}(3L^2 - 4x^2)^2 \end{cases} \]
**Eq 412-8**

\[ R = V \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots = \frac{P}{2} \]

\[ M_m \quad \text{[at load]} \quad \ldots \ldots \ldots \ldots = P L \frac{4}{E I} \]

\[ M_x \quad \text{[at x < L/2]} \quad \ldots \ldots \ldots \ldots = \frac{P x}{2} \]

\[ D_m \quad \text{[at load]} \quad \ldots \ldots \ldots \ldots = \frac{P L^3}{48 E I} \]

\[ D_x \quad \text{[at x < L/2]} \quad \ldots \ldots \ldots \ldots = \frac{P x}{48 E I} \left( 3L^2 - 4x^2 \right) \]

**Eq 412-9**

\[ R_1 = V_1 \quad \text{[max. @ a < b]} \quad \ldots \ldots = \frac{P b}{L} \]

\[ R_2 = V_2 \quad \text{[max. @ a > b]} \quad \ldots \ldots = \frac{P a}{L} \]

\[ M_m \quad \text{[at load]} \quad \ldots \ldots \ldots \ldots = \frac{P a b}{L} \]

\[ M_x \quad \text{[at x < a]} \quad \ldots \ldots \ldots \ldots = \frac{P b x}{L} \]

\[ M_x \quad \text{[at a < x < L]} \quad \ldots \ldots \ldots \ldots = \frac{P a}{L} (L - x) \]

\[ D_m \quad \text{[at x = \frac{a(a + 2b)}{3}]} \quad \text{[at a > b]} \quad \ldots \ldots = \frac{P a b}{6} \left( a + 2b \right) \left( a + 3a + 2b \right) \]

\[ D_x \quad \text{[at load]} \quad \ldots \ldots \ldots \ldots = \frac{P a b^2}{3 E I} \]

\[ D_x \quad \text{[at x < a]} \quad \ldots \ldots \ldots \ldots = \frac{P b x}{6 E I} \left( L^2 - b^2 - x^2 \right) \]

\[ D_x \quad \text{[at a < x < L]} \quad \ldots \ldots \ldots \ldots = \frac{P a}{6 E I} \left( L - x \right) \left( -a^2 + 2xL - x^2 \right) \]

2 Equal loads

**Eq 412-10 a = b = c = L/3**

\[ R = V \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots = P \]

\[ M_m \quad \text{[between loads]} \quad \ldots = \frac{P L}{3} \]

\[ M_x \quad \text{[at x < L/3]} \quad \ldots \ldots = \frac{P x}{2} \]

\[ D_m \quad \text{[at center]} \quad \ldots \ldots \ldots \ldots = \frac{23P L^3}{64 E I} \]

\[ D_x \quad \text{[at x < a]} \quad \ldots \ldots \ldots \ldots = \frac{P x}{6 E I} \left( \frac{2b^2}{3} - x^2 \right) \]

\[ D_x \quad \text{[at a < x < (L - a)]} \ldots \ldots \ldots \ldots = \frac{P L}{18 E I} \left( 3Lx - 3x^2 - L^2/9 \right) \]

**Eq 412-12 a ≠ b ≠ c**

\[ R_1 = V_1 \quad \text{[max. @ a < b]} \quad \ldots \ldots = \frac{P}{L} (L - a + b) \]

\[ R_2 = V_2 \quad \text{[max. @ a > b]} \quad \ldots \ldots = \frac{P}{L} (L - b + a) \]

\[ V_3 \quad \ldots \ldots \ldots \ldots \ldots = \frac{P}{L} (b - a) \]

\[ M_1 \quad \text{[max. @ a > b]} \quad \ldots \ldots = R_1 a \]

\[ M_2 \quad \text{[max. @ a < b]} \quad \ldots \ldots = R_2 b \]

\[ M_x \quad \text{[at x < a]} \quad \ldots \ldots \ldots \ldots = R_1 x \]

\[ M_x \quad \text{[at a < x < (L - b)]} \ldots \ldots \ldots \ldots = R_1 x - P(x - a) \]

\[ M_x \quad \text{[at (L - b) < x < L]} \quad \ldots \ldots \ldots \ldots = R_2 (L - x) \]

**NOTE:** For deflections use superposition of 2 single concentrated loads.
2 Unequal loads

Eq 412-13 \( P_1 \neq P_2 \)

\[
\begin{align*}
R_1 &= V_1 = P_1 (L - a) + P_2 b \\
R_2 &= V_2 = P_1 a + P_2 (L - b) \\
V_1 &= V_2 = R_1 - P_1 \\
M_1 &= \text{max. } R_1 < P_1 = R_1 a \\
M_2 &= \text{max. } R_2 < P_2 = R_2 b \\
M_x &= \text{max. } 0 < x < a = R_1 x \\
M_x &= \text{max. } a < x < (L - b) = R_1 x - P_1 (x - a) \\
\end{align*}
\]

NOTE: For deflections use superposition of 2 single concentrated loads.

Eq 412-14

\[
\begin{align*}
R &= V_1 = \frac{3P}{2} \\
V_2 &= \frac{P}{2} \\
M_x &= \text{max. } 0 < x < L/4 = \frac{3P x}{2} \\
M_x &= \text{max. } L/4 < x < L/2 = \frac{P x}{2} + \frac{P L}{4} \\
M_1 &= \text{max. } 0 < x = L/4 = \frac{3P L}{8} \\
M_n &= \text{max. } 0 < x = L/2 = \frac{P L}{2} \\
D_m &= \text{max. } 0 < x = L/2 = \frac{19P L^3}{384 E I} \\
\end{align*}
\]

Eq 412-15

\[
\begin{align*}
R_1 &= V_1 = P_1 (L - a) + P_2 (c + d) + P_3 d \\
R_2 &= V_2 = P_1 a + P_2 (a + b) + P_3 (L - d) \\
V_1 &= V_2 = R_1 - P_1 \\
V_2 &= V_3 = R_1 - P_1 - P_2 \\
M_1 &= \text{max. } R_1 \geq R_3 = M_1 \\
M_2 &= \text{max. } P_1 + P_2 \geq R_1, P_2 + P_3 \geq R_2 = M_2 \\
M_3 &= \text{max. } P_3 \geq R_2 = M_3 \\
\end{align*}
\]

NOTE: For deflections use superposition of 3 single concentrated loads.

Eq 412-16

\[
\begin{align*}
R &= V_1 = 2P \\
V_2 &= P \\
M_x &= \text{max. } 0 < x < L/5 = 2P x \\
M_x &= \text{max. } L/5 < x < 2L/5 = P x + PL/5 \\
M_1 &= \frac{2P L}{5} \\
M_n &= \text{max. } 0 < x = L/2 = \frac{3P L}{5} \\
D_m &= \text{max. } 0 < x = L/2 = \frac{3.78 P L^3}{60 E I} \\
\end{align*}
\]


**Eq 412-17**

\[ R_1 = V_1 m \left( \theta \ x = 0 \right) \ldots \ldots \ldots = P \]

\[ M_m \left( \theta \ load, \ \theta \ x = \frac{L}{2} \right) \ldots \ldots = \frac{P L}{4} \]

**Eq 412-18**

\[ R_1 = V_1 m \left( \theta \ x = 0 \right) \ldots \ldots \ldots \ldots \ldots = P \left( 2 - \frac{a}{L} \right) \]

\[
\begin{align*}
@ a < \frac{.586 L}{\text{load 1 @ x = } \frac{1}{2} (L - \frac{a}{2})} & \ldots \ldots = \frac{P}{2L} \left( L - \frac{a}{2} \right)^2 \\
@ a > .586 L \text{ with one load at center of span} & \ldots = \frac{P L}{4} \\
\end{align*}
\]

**Eq 412-19**

\[ R_1 = V_1 m \left( \theta \ x = 0 \right) \ldots \ldots \ldots \ldots = P_1 + P_2 \frac{L - a}{L} \]

\[
\begin{align*}
\begin{cases}
\text{under } P_1, @ x = \frac{1}{2} L - \frac{P_2 a}{P_1 + P_2} & = \left( P_1 + P_2 \right) \frac{x^2}{L} \\
\text{M_m may be with } P_1 \text{ at center of span and } P_2 \text{ off span} & = \frac{P_1 L}{4} \\
\end{cases}
\end{align*}
\]

Maximum shear is at one support when one of the loads is at that support. With several moving loads, locate them for maximum shear by trial.

Maximum moment is under one of the loads when that load is as far from one support as the center of gravity of all the moving loads on the beam is from the other support.

M_m is at P_1 when x = b, and when the span center line is midway between the center of gravity of loads and the nearest concentrated load.

**Cantilever Beams**

**Eq 412-20**

\[ R = V \ldots \ldots \ldots \ldots \ldots = w L \]

\[ V_x \ldots \ldots \ldots \ldots \ldots = w x \]

\[ M_m \left[ \theta \ fixed \ end \right] \ldots \ldots \ldots = -\frac{w L^2}{2} \]

\[ M_x \ldots \ldots \ldots \ldots \ldots = -\frac{w x^2}{2} \]

\[ M_m \left[ \theta \ free \ end \right] \ldots \ldots \ldots = \frac{w L^4}{8 E I} \]

\[ D_x \ldots \ldots \ldots \ldots \ldots = -\frac{w}{24 E I} \left( x^8 - 4 L^3 x + 3 L^5 \right) \]

For partial load from support to distance KL, substitute KL for L. Measure x from left end of KL. D_m then becomes D_KL.
Overhang Beams

Eq 412-21

\[ W = \frac{WL}{2} \]
\[ R = V = \frac{WL}{2} \]
\[ V_x = \frac{wx^2}{2L} \]
\[ M_x \left[ \text{@ fixed end} \right] = \frac{-WL^2}{6} \]
\[ M_x = \frac{-wx^3}{6L} \]
\[ D_m \left[ \text{@ free end} \right] = \frac{wL^4}{30EI} \]
\[ D_x = \frac{w}{120EI}(x^5 - 5L^4x + 4L^5) \]

Eq 412-22 \( a = 0 \) (load at free end)

\[ R = V = P \]
\[ M_n \left[ \text{@ fixed end} \right] = -PL \]
\[ M_x = P \]
\[ D_x = \frac{P}{6EI}(2L^3 - 3L^2x + x^3) \]

Eq 412-23 \( a \neq 0 \)

\[ R = V \left[ \text{@ } a < x < L \right] = P \]
\[ M_m \left[ \text{@ fixed end} \right] = -Pb \]
\[ M_x \left[ \text{@ x > a} \right] = -P(x - a) \]
\[ D_m \left[ \text{@ free end} \right] = \frac{Pb^2}{6EI}(3L - b) \]
\[ D_x \left[ \text{@ load} \right] = \frac{Pb^3}{3EI} \]
\[ D_x \left[ \text{@ } x < a \right] = \frac{Pb^2}{6EI}(3L - 3x - b) \]
\[ D_x \left[ \text{@ x > a} \right] = \frac{P(L - x)^2}{6EI}(3b - L + x) \]

Eq 412-24

\[ R_1 = V_1 = \frac{wa}{2L}(L^2 - a^2) \]
\[ R_2 = V_2 + V_3 = \frac{wa}{2L}(L + a)^2 \]
\[ V_2 = wa \]
\[ V_3 = \frac{wa}{2L}(L^2 + a^2) \]
\[ V_x = wa \]
\[ V_{x1} = -\frac{wa^2}{2} \]
\[ M_x \left[ \text{@ x = \frac{L}{2}(1 - \frac{a^2}{L^2})} \right] = -\frac{wa^2}{8L^2}(L + a)^2(L - a)^2 \]
\[ M_x \left[ \text{@ R_2} \right] = -\frac{wa^2}{2} \]
\[ M_x = \frac{wa}{2L}(L^2 - a^2 - xL) \]
\[ M_{x1} = -\frac{wa}{2}(a - x_1)^2 \]
\[ D_x = \frac{wa}{24EI}(L^2 - 2L^2x^2 + Lx^3 - 2a^2L^2 + 2a^2x^2) \]
\[ D_{x1} = \frac{wa}{24EI}(6a^2L - L^3 + 6a^2x_1 - 4ax_1^2 + x_1^3) \]
Eq 412-25

\[ R_1 = V_1 = \frac{-wa^2}{2L} \]
\[ R_2 = V_1 + V_2 = \frac{-wa(2L + a)}{2L} \]
\[ V_2 = wa \]
\[ Vx_1 = wa(a - x_1) \]
\[ Mm = \left[ \theta R_2 \right] = \frac{-wa^2}{2} \]
\[ Mx = \frac{-wa^2x}{2L} \]
\[ Mx_1 = \frac{-w}{3}(a - x_1)^2 \]
\[ Dnx = \left[ \theta x = \frac{L}{3} \right] = \frac{-wa^2L}{18\sqrt{3}EI} = \frac{-0.03208wa^2L^2}{EI} \]
\[ Dnx_1 = \left[ \theta x_1 = a \right] = \frac{-wa^3}{24EI}(4L + 3a) \]
\[ Dx = \frac{-wa^2x}{12EI}(L^2 - x^2) \]
\[ Dx_1 = \frac{-wa^2x}{24EI}(4a^2L + 6ax_1 - 4ax_1^2 + x_1^3) \]

Eq 412-26

\[ R = V = \frac{wL}{2} \]
\[ Vx = \frac{w(L/2 - x)}{2} \]
\[ Mm = \left[ \text{at center} \right] = \frac{wL^3}{8} \]
\[ Mx = \frac{wL^2}{2}(L - x) \]
\[ Dm = \left[ \text{at center} \right] = \frac{5wL^4}{384EI} \]
\[ Dx = \frac{wL}{24EI}(L^3 - 2Lx^2 + x^3) \]
\[ Dx_1 = \frac{-wL^3x_1}{24EI} \]

Eq 412-27

\[ R_1 = \frac{wL(L - 2c)}{2b} \]
\[ R_2 = \frac{wL(L - 2a)}{2b} \]
\[ V_1 = wa \]
\[ V_2 = R_1 - V_1 \]
\[ V_3 = R_2 - V_2 \]
\[ V_4 = wc \]
\[ Vx_1 = V_1 - Vx_1 \]
\[ Vx = \left[ \theta x < L \right] = R_1 - w(a + x_1) \]
\[ Vm = \left[ \theta a < c \right] = R_2 - wc \]
\[ M_1 = \frac{-wa^2}{2} \]
\[ M_2 = \frac{-wc^2}{2} \]
\[ M_3 = \frac{-wa^2}{2} \]
\[ M_4 = R_1 \left( \frac{KL}{w} - a \right) \]
\[ Mx_1 = \left[ \text{max. } \theta x = \frac{R_1}{w} - a \right] = R_1x - \frac{w(a + x)^2}{2} \]
\[ Mx_2 = \frac{-wa^2x}{2} \]
Multi-Span Beams

Eq 412-28
\[ R_1 = V_1 \quad \text{[max. } a < b \text{]} \quad \Rightarrow \quad \frac{Pb}{L} \]
\[ R_2 = V_2 \quad \text{[max. } a > b \text{]} \quad \Rightarrow \quad \frac{Pa}{L} \]
\[ M_n \quad \text{[at load]} \quad \Rightarrow \quad \frac{Pab}{L} \]
\[ M_x \quad \text{[at } x \text{]} \quad \Rightarrow \quad \frac{Pbx}{L} \]
\[ D_n \quad \text{[at } x = \frac{a(a + 2b)}{3} \text{]} \quad \Rightarrow \quad \frac{Pab(a + 2b)\sqrt{3a(a + 2b)}}{27EI} \]
\[ D_a \quad \text{[at load]} \quad \Rightarrow \quad \frac{PPositionError \_2 \text{b}^2}{3EI} \]
\[ D_x \quad \text{[at } x \text{]} \quad \Rightarrow \quad \frac{Pbx^3}{6EIL} \]
\[ D_x \quad \text{[at } x > a \text{]} \quad \Rightarrow \quad \frac{P(a - x)(2Lx - x^2 - a^2)}{6EIL} \]
\[ D_{x1} \quad \Rightarrow \quad \frac{Pbx(L + a)}{6EIL} \]

Eq 412-29
\[ R_1 = V_1 \quad \Rightarrow \quad -\frac{Pa}{L} \]
\[ R_2 = V_1 + V_2 \quad \Rightarrow \quad \frac{P}{L}(L + a) \]
\[ V_2 \quad \Rightarrow \quad P \]
\[ M_n \quad \text{[at } R_2 \text{]} \quad \Rightarrow \quad -\frac{Pa}{L} \]
\[ M_x \quad \text{[between supports]} \quad \Rightarrow \quad -\frac{Pax}{L} \]
\[ M_{x1} \quad \text{[for overhang]} \quad \Rightarrow \quad -P(a - x_1) \]
\[ D_n \quad \text{[at } x = \frac{L}{4} \text{]} \quad \Rightarrow \quad -0.06415 \frac{Pb^2}{EI} \]
\[ D_n \quad \text{[at } x_1 = a \text{]} \quad \Rightarrow \quad \frac{Pb^2}{3EI}(L + a) \]
\[ D_x \quad \text{[between supports]} \quad \Rightarrow \quad -\frac{Pax}{6EIL}(L^2 - x^2) \]
\[ D_{x1} \quad \text{[for overhang]} \quad \Rightarrow \quad \frac{Pb}{6EIL}(2al + 3ax_1 - x_1^2) \]

Eq 412-30
\[ R_1 = V_1 = R_3 = V_3 \quad \Rightarrow \quad \frac{3wl}{8} \]
\[ R_2 \quad \Rightarrow \quad \frac{10wl}{8} \]
\[ V_2 = V_3 \quad \Rightarrow \quad \frac{5wl}{8} \]
\[ M_n \quad \Rightarrow \quad -\frac{wl^2}{8} \]
\[ M_1 \quad \text{[at } x = \frac{3L}{8} \text{]} \quad \Rightarrow \quad \frac{9wl^2}{128} \]
\[ M_x \quad \text{[at } x < L \text{]} \quad \Rightarrow \quad \frac{3wl^2}{8} - \frac{wx^2}{2} \]
\[ D_n \quad \text{[at approx. 0.46L]} \quad \Rightarrow \quad \frac{wl^2}{185EI} \]
Eq 412-31

\[ R_1 = V_1 = \frac{7}{16}wL \]
\[ R_2 = V_2 + V_3 = \frac{5}{8}wL \]
\[ R_3 = V_3 = -\frac{1}{16}wL \]
\[ V_2 = -\frac{9}{16}wL \]
\[ M_x = \left[ \theta x = \frac{7}{16}L \right] = \frac{49}{512}wL^2 \]
\[ M_1 = \left[ \theta > \frac{L}{2} \right] = -\frac{1}{16}wL^2 \]
\[ M_x = \left[ \theta < L \right] = \frac{wL^2}{16}(7L - 8x) \]
\[ D_m = \left[ \theta = 0.472L \right] = -\frac{0.092wL^2}{EI} \]

Eq 412-32

\[ R_1 = V_1 = \frac{M_1}{L_1} + \frac{wL_1}{2} \]
\[ R_2 = V_2 = \frac{M_1}{L_2} + \frac{wL_2}{2} \]
\[ R_3 = V_3 = \frac{M_1}{L_3} + \frac{wL_3}{2} \]
\[ V_2 = wL_1 - R_1 \]
\[ V_3 = wL_2 - R_3 \]
\[ M_1 = \frac{wL_3^3 + wL_1^3}{8(L_1 + L_2)} \]
\[ M_{x1} = \left[ \theta < L \right] \text{max. } \theta x = \frac{R_3}{w} \]
\[ R_2x = \frac{wL_2^2}{2} \]
\[ M_{x1} = \left[ \theta < L \right] \text{max. } \theta x = \frac{R_3}{w} \]
\[ R_3x_1 = \frac{wL_3^2}{2} \]

Eq 412-33

\[ V_1 = V_2 = V_3 = V_4 = \frac{4wL}{10} \]
\[ R_2 = R_3 = \frac{11wL}{10} \]
\[ V_2 = V_5 = \frac{6wL}{10} \]
\[ V_3 = V_4 = \frac{wL}{2} \]
\[ M_x = \left[ \theta < L \right] \text{max. } \theta x = \frac{wLx}{10} \]
\[ M_{x1} = \left[ \theta < L \right] \text{max. } \theta x = \frac{wLx}{10} \]
\[ D_1 = \frac{D_1}{EI} = \frac{4wL^4}{581EI} \]
\[ D_2 = \frac{D_2}{EI} = \frac{wL^4}{1920EI} \]

Max. Pos. Nom. \[ M_3 = \left[ \theta x = \frac{4L}{10} \right] = \frac{2wL^3}{25} \]

Max. Neg. Nom. \[ M_2 \left[ \theta x = 1 \right] = -\frac{wL^2}{10} \]
**Eq 412-34**

\[
R_1 = V_1 = 0.383wL \\
R_2 = 1.200wL \\
R_3 = 0.650wL \\
R_4 = -0.032wL \\
V_2 = 0.617wL \\
V_3 = 0.583wL \\
V_5 = 0.417wL \\
V_6 = -0.033wL \\
\text{Max. Neg. Mom., } M_2 \quad [\theta \times = L] = -0.116wL^2 \\
\text{Max. Pos. Mom., } M_1 \quad [\theta \times = 0.383L] = 0.0735wL^2 \\
M_x \quad [\theta \times < L \text{ max. } \theta \times = 0.383L] = 0.383wLx - \frac{wL^2}{2} \\
M_{x1} \quad [\theta \times_1 < L \text{ max. } \theta \times_1 = 0.583L] = 0.583wLx_1 - \frac{wL^2}{2} - \frac{0.116wL^2}{2} \\
M_{x2} \quad [\theta \times_2 < L] = -0.032wLx_2 \\
D_m \quad [\theta \times = 0.430L] = 0.0059wL^2 \\
\]

**Eq 412-35**

\[
R_1 = R_4 = V_1 = V_4 = 0.450wL \\
R_2 = R_3 = V_2 = V_3 = 0.550wL \\
\text{Max. Neg. Mom., } M_2 \quad [\theta \times = L] = -0.056wL^2 \\
\text{Max. Pos. Mom., } M_1 \quad [\theta \times = 0.450L] = 0.1013wL^2 \\
M_x \quad [\theta \times < L] = 0.450wLx - \frac{wL^2}{2} \\
D_m \quad [\theta \times = 0.479L] = 0.0099wL^2 \\
\]

**Eq 412-36**

\[
R_1 = R_5 = V_1 = V_6 = 0.393wL \\
R_2 = R_4 = 1.143wL \\
R_3 = 0.928wL \\
V_2 = V_7 = 0.536wL \\
V_3 = V_5 = 0.607wL \\
V_4 = V_6 = 0.464wL \\
V_5 = -0.0714wL^2 \\
M_3 \quad \text{Max. Pos. Mom., } M_1 \quad [\theta \times = 0.393L] = 0.393wLx - \frac{wL^2}{2} \\
M_{x1} \quad [\theta \times_1 < L \text{ max. } \theta \times_1 = 0.536L] = 0.536wLx_1 - \frac{0.1071wL^2}{2} - \frac{0.0065wL^2}{2} \\
D_m \quad [\theta \times = 0.440L] = 0.0065wL^2 \\
M_4 = 0.036wL^2 \\
\]
Eq 412-37

\[ R_1 = V_1 = 0.380 \text{wL} \]
\[ R_2 = V_2 = 1.223 \text{wL} \]
\[ R_3 = R_4 = 0.357 \text{wL} \]
\[ R_5 = 0.598 \text{wL} \]
\[ R_6 = V_6 = 0.442 \text{wL} \]
\[ V_2 = 0.620 \text{wL} \]
\[ V_3 = 0.603 \text{wL} \]
\[ V_4 = 0.397 \text{wL} \]
\[ V_5 = 0.040 \text{wL} \]
\[ V_6 = 0.558 \text{wL} \]
\[ M_x [\theta x < L \max, \theta x = 0.380L] = 0.380wLx - \frac{wx^2}{2} \]
\[ M_{x1} [\theta x < L \max, \theta x = 0.603L] = 0.603wLx_1 - \frac{0.1205wL^2}{2} - \frac{wx_1^2}{2} \]
\[ M_{x2} [\theta x < L \max, \theta x = L] = -0.046Lx_2 - 0.0179wL^2 \]
\[ M_{x3} [\theta x < L \max, \theta x = 0.442L] = 0.442wLx_3 - \frac{wx_3^2}{2} \]
\[ D_m [\theta x = 0.475L] = \frac{0.0094wL^4}{EI} \]

Eq 412-38

\[ R_1 = V_1 = 0.446wL \]
\[ R_2 = V_4 = 0.572wL \]
\[ R_3 = V_3 = 0.464wL \]
\[ R_5 = -0.054wL \]
\[ V_2 = 0.554wL \]
\[ V_3 = 0.018wL \]
\[ V_4 = 0.482wL \]
\[ V_5 = 0.518wL \]
\[ V_6 = V_7 = 0.056wL \]
\[ M_x [\theta x < L \max, \theta x = 0.446L] = 0.446wLx - \frac{wx^2}{2} \]
\[ M_{x1} [\theta x < L \max, \theta x = 0] = 0.018wLx_1 - 0.0536wL^2 \]
\[ M_{x2} [\theta x < L \max, \theta x = 0.482L] = 0.482wLx_2 - 0.0357wL^2 - \frac{wx_2^2}{2} \]
\[ M_{x3} [\theta x < L \max, \theta x = L] = -0.054wLx_3 \]
\[ D_m [\theta x = 0.477L] = \frac{0.0097wL^4}{EI} \]

Eq 412-39

\[ R_1 = V_1 = \frac{13}{32} \text{P} \]
\[ R_2 = V_2 + V_3 = \frac{11}{16} \text{P} \]
\[ R_3 = V_3 = \frac{3}{32} \text{P} \]
\[ V_2 = -\frac{19}{32} \text{P} \]
\[ M_x [\theta load] = \frac{13}{64} \text{PL} \]
\[ M_1 [\theta R_2] = -\frac{3}{32} \text{PL} \]
Eq 412-40
\[ R_1 = V_1 = \frac{P_a}{4L^3}(4L^2 - a(L + a)) \]
\[ R_2 = V_2 + V_3 = \frac{P_a}{2L^3}(2L^2 + b(L + a)) \]
\[ R_3 = V_3 = -\frac{P_a b}{4L^3}(L + a) \]
\[ V_2 = \frac{P_a}{4L^3}(4L^2 + b(L + a)) \]
\[ M_m \text{ [@ load]} = \frac{P_a b}{4L^2}(L + a) \]

Eq 412-41
\[ R_1 = V_1 = R_3 = V_3 = \frac{5P}{16} \]
\[ R_2 = 2V_2 = \frac{11P}{8} \]
\[ V_2 = P - R_1 = \frac{11P}{16} \]
\[ M_m = \frac{3PL}{16} \]
\[ M_1 \] [\( b < a \)] = \[ R_1 x \]
\[ M_1 \] [\( a < x < L \)] = \[ R_1 x - P(x - L/2) \]

Eq 412-42
\[ R_1 = V_1 = \frac{M_1}{L_1} + \frac{P_1}{2} \]
\[ R_2 = P_1 + P_2 - R_1 - R_3 \]
\[ R_3 = V_4 = \frac{M_2}{L_2} + \frac{P_2}{2} \]
\[ V_2 = P_1 - R_1 \]
\[ V_3 = P_2 - R_3 \]
\[ M_2 = R_1 a \]
\[ M_3 = R_2 b \]
\[ M_3 = \frac{3}{16}(\frac{P_1 L_1 ^2 + P_2 L_2 ^2}{L_1 + L_2}) \]
\[ M_4 = [b < a < L] = R_1 x \]
\[ M_4 = [a < x < L] = R_1 x - P(x - a) \]
\[ M_4 = [b < x_1 < L] = R_3 x_1 \]
\[ M_4 = [b < x_1 < L] = R_3 x_1 - P(x_1 - b) \]

Eq 412-43
\[ R_1 = V_1 = \frac{M_1}{L} + \frac{P_1 b_1}{L} \]
\[ R_2 = 2R_1 + P_2 + P_1(L + b_1) \]
\[ R_3 = M_2 + P_2 b_2 \]
\[ V_2 = R_1 - P_1 \]
\[ V_3 = R_2 - V_2 \]
\[ V_4 = R_3 - V_3 \]
\[ V_5 = R_4 - V_5 \]
\[ M_1 = -\frac{4P_1 a_1 b_1(L + a_1) - P_2 c_1 c_2(7L - 5c_1) + P_2 b_2 a_2(L + a_2)}{15L^2} \]
\[ M_2 = \frac{P_1 a_1 b_1(L + a_1) - P_2 c_1 c_2(2L + 5c_1) - 4P_2 b_2 a_2(L + a_2)}{15L^2} \]
\[ M_3 = R_1 a_1 \]
\[ M_4 = M_1 + V_3 c_1 \]
\[ M_5 = R_4 a_2 \]
Eq 412-44

\[ R_1 = V_1 \quad \ldots \quad \frac{M_1}{L} - \frac{WL}{2} \]

\[ R_2 = V_4 \quad \ldots \quad 2WL + P - R_1 - R_3 \]

\[ R_3 = V_4 \quad \ldots \quad \frac{M_1}{L} + \frac{Pa}{L} + \frac{WL}{2} \]

\[ V_3 = 3WL - R_1 \]

\[ V_3 = 3WL + P - R_3 \]

\[ M_1 \quad \ldots \quad -\frac{WL^2}{8} - \frac{Pb(L^2 - b^2)}{4L^2} \]

\[ M_2 \quad [\theta \times = \frac{R_2}{W^2}] \quad \ldots \quad R_2 x = \frac{Wx_2}{2} \]

\[ M_3 \quad [\theta \times = \frac{R_3}{W^2} \text{ when } R_3 < \text{wb}] \quad R_3 x_3 = \frac{Wx_3}{2} \]

Eq 412-45

\[ R_1 = V_1 \quad \ldots \quad \frac{M_1}{L} + \frac{P_2b}{L} + \frac{WL_2}{2} \]

\[ R_2 = V_4 \quad \ldots \quad \frac{M_1}{L} + \frac{P_2c}{L} + \frac{WL_2}{2} \]

\[ R_3 = V_4 \quad \ldots \quad \frac{M_1}{L} + \frac{P_2c}{L} + \frac{WL_2}{2} \]

\[ V_2 = WL_1 + P_1 - R_1 \]

\[ V_3 = WL_2 + P_2 - R_3 \]

\[ M_1 \quad \ldots \quad \left[\frac{-4P_1L_1^2\left(\frac{a_1}{L_1} - \frac{a_2}{L_1}\right) + 4P_2L_2^2\left(\frac{a_1}{L_2} - \frac{a_2}{L_2}\right) + WL_1 + WL_2}{8(L_1 + L_2)}\right] \]

\[ M_2 \quad [\theta \times = \frac{R_1}{W^2} \text{ when } R_1 < \text{wa}] \quad \ldots \quad R_3 x_3 = \frac{Wx_3}{2} \]

\[ M_3 \quad [\theta \times = \frac{R_3}{W^2} \text{ when } R_3 \leq \text{wg}] \quad \ldots \quad R_3 x_3 = \frac{Wx_3}{2} \]

Eq 412-46

\[ R_1 = V_1 \quad \ldots \quad \frac{WL}{2} + \frac{M_1}{L} + \frac{P_1b_1}{L} \]

\[ R_2 = V_4 \quad \ldots \quad 2WL + \frac{M_1}{L} + \frac{P_2c_2}{L} + \frac{P_1(L + b_1)}{L} \]

\[ R_3 = V_4 \quad \ldots \quad 2WL + \frac{M_1}{L} + \frac{P_2c_3}{L} + \frac{P_3(L + b_2)}{L} \]

\[ R_4 = V_5 \quad \ldots \quad \frac{WL}{2} + \frac{M_1}{L} + \frac{P_3b_2}{L} \]

\[ V_2 = P_1 + WL - R_3 \]

\[ V_3 = R_2 - V_2 \]

\[ V_4 = R_3 - V_3 \]

\[ V_5 = R_3 - V_5 \]

\[ M_1 \quad \ldots \quad -\frac{4P_1a_1b_1(L + a_1)}{15L^2} - \frac{P_2c_1c_2(2L + 5c)}{15L^2} - \frac{P_3b_2a_2(L + a_2)}{15L^2} - \frac{WL^2}{10} \]

\[ M_2 \quad \ldots \quad \frac{P_1a_1b_1(L + a_1)}{15L^2} - \frac{P_2c_1c_2(2L + 5c)}{15L^2} - \frac{P_3b_2a_2(L + a_2)}{15L^2} - \frac{WL^2}{10} \]

\[ M_3 \quad \ldots \quad \frac{R_3a_1}{2} \]

\[ M_4 \quad \ldots \quad \frac{M_1 + V_3c_1}{2} \]

\[ M_5 \quad \ldots \quad \frac{R_4a_2}{2} \]
### Fixed-End Beams

#### Eq 412-47

<table>
<thead>
<tr>
<th>Both Ends Fixed</th>
<th>$f_n$</th>
<th>$f_b$</th>
<th>$P_n$</th>
<th>$P_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$wL^2/12$</td>
<td>$wL^2/12$</td>
<td>$wL/2$</td>
<td>$wL/2$</td>
</tr>
<tr>
<td>2</td>
<td>$w[123L(3L^2+5L-93)]/8$</td>
<td>$w[123L(3L^2+5L-93)]/8$</td>
<td>$M_N-M_N\cdot v_b(L+a)$</td>
<td>$M_N-M_N\cdot v_b(2a)$</td>
</tr>
<tr>
<td>3</td>
<td>$wL^2/12[L(3L^2+5L-93)/8]$</td>
<td>$wL^2/12[L(3L^2+5L-93)/8]$</td>
<td>$M_N-M_N\cdot v_b(L-a)$</td>
<td>$M_N-M_N\cdot v_b(2a)$</td>
</tr>
<tr>
<td>4</td>
<td>$wL^2/6$</td>
<td>$wL^2/6$</td>
<td>$wL/2$</td>
<td>$wL/2$</td>
</tr>
</tbody>
</table>

#### Near End Hinged, $M_A = 0$

<table>
<thead>
<tr>
<th>Fig #</th>
<th>$f_n$</th>
<th>$f_b$</th>
<th>$P_n$</th>
<th>$P_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0</td>
<td>$wL^2[120L^2 - 3a^2]$</td>
<td>$-\frac{wL^2}{L}(L - a/3)$</td>
<td>$-\frac{wL^2}{L}$</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>$wL^2[30L^2 - 3a^2]$</td>
<td>$-\frac{wL^2}{L}(L - 2a/3)$</td>
<td>$-\frac{wL^2}{L}$</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>$wL^2[2L^2 - a^2]$</td>
<td>$-\frac{wL^2}{L}(L - 4a/3)$</td>
<td>$-\frac{wL^2}{L}$</td>
</tr>
</tbody>
</table>

#### Far End Hinged, $M_B = 0$

<table>
<thead>
<tr>
<th>Fig #</th>
<th>$f_n$</th>
<th>$f_b$</th>
<th>$P_n$</th>
<th>$P_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$wL^2/8$</td>
<td>0</td>
<td>$5wL/8$</td>
<td>$3wL/8$</td>
</tr>
<tr>
<td>2</td>
<td>$wL^2[2L^2 + 2b^2 - 2c^2 - 2(ab - b^2)]$</td>
<td>0</td>
<td>$P_N + (wL/3) + (wL/3)$</td>
<td>$-\frac{wL}{L}$</td>
</tr>
<tr>
<td>3</td>
<td>$wL^2[2L - a^2]$</td>
<td>0</td>
<td>$P_N = v_b(L - a/3)$</td>
<td>$-\frac{wL}{L}$</td>
</tr>
<tr>
<td>4</td>
<td>$wL^2/[2L - 2a]$</td>
<td>0</td>
<td>$P_N = v_b(L - 2a/3)$</td>
<td>$-\frac{wL}{L}$</td>
</tr>
<tr>
<td>5</td>
<td>$wL^2/15$</td>
<td>0</td>
<td>$3wL/5$</td>
<td>$wL/10$</td>
</tr>
<tr>
<td>6</td>
<td>$wL^2[120L^2 + 15ac + 3a^2]$</td>
<td>0</td>
<td>$P_N = v_b(L - a/3)$</td>
<td>$-\frac{wL}{L}$</td>
</tr>
<tr>
<td>7</td>
<td>$wL^2[120L^2 + 45ac + 12a^2]$</td>
<td>0</td>
<td>$P_N = v_b(L - 2a/3)$</td>
<td>$-\frac{wL}{L}$</td>
</tr>
<tr>
<td>8</td>
<td>$wL^2/10$</td>
<td>0</td>
<td>$P_N = v_b(L - a/2)$</td>
<td>$-\frac{wL}{L}$</td>
</tr>
<tr>
<td>9</td>
<td>$P_N = \frac{5L}{L}$</td>
<td>0</td>
<td>$P_N = P_b$</td>
<td>$-\frac{wL}{L}$</td>
</tr>
<tr>
<td>10</td>
<td>$P_N = (L - a)$</td>
<td>0</td>
<td>$P_N = v_b(L - a/2)$</td>
<td>$-\frac{wL}{L}$</td>
</tr>
<tr>
<td>11</td>
<td>$15P_L/16$</td>
<td>0</td>
<td>$P_N = v_b(L - a/2)$</td>
<td>$-\frac{wL}{L}$</td>
</tr>
</tbody>
</table>
Table 631-1. Insulation values.
From 1981 ASHRAE Handbook of Fundamentals. Values do not include surface conditions unless noted otherwise. All values are approximate.

<table>
<thead>
<tr>
<th>Material</th>
<th>Per inch (approximate)</th>
<th>For thickness listed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batt and blanket insulation</td>
<td>3.00-3.80*</td>
<td></td>
</tr>
<tr>
<td>Glass or mineral wool, fiberglass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fill-type insulation</td>
<td>3.13-3.70</td>
<td></td>
</tr>
<tr>
<td>Glass or mineral wool</td>
<td>2.50-3.00</td>
<td></td>
</tr>
<tr>
<td>Vermiculite</td>
<td>2.20</td>
<td></td>
</tr>
<tr>
<td>Shavings or sawdust</td>
<td>2.22</td>
<td></td>
</tr>
<tr>
<td>Hay or straw, 20&quot;</td>
<td>30+</td>
<td></td>
</tr>
<tr>
<td>Rigid insulation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exp. polystyrene, extruded, plain</td>
<td>6.25</td>
<td></td>
</tr>
<tr>
<td>molded beads, 1 pcf</td>
<td>4.55</td>
<td></td>
</tr>
<tr>
<td>molded beads, over 1 pcf</td>
<td>4.20</td>
<td></td>
</tr>
<tr>
<td>Expanded rubber</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expanded polyurethane, aged</td>
<td>5.00</td>
<td></td>
</tr>
<tr>
<td>Glass fiber</td>
<td>4.00</td>
<td></td>
</tr>
<tr>
<td>Wood or cane fiberboard</td>
<td>2.50</td>
<td></td>
</tr>
<tr>
<td>Polyisocyanurate</td>
<td>7.04</td>
<td></td>
</tr>
<tr>
<td>Foamed-in-place insulation</td>
<td>7.04</td>
<td></td>
</tr>
<tr>
<td>Polyurethane</td>
<td>6.00</td>
<td></td>
</tr>
<tr>
<td>Building materials</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete, solid</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>Concrete block, 3 hole, 8&quot;</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>lightweight aggregate, 8&quot;</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>lightweight, cores insulated</td>
<td>5.03</td>
<td></td>
</tr>
<tr>
<td>Brick, common</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Metal siding</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>hollow-backed</td>
<td>0.61</td>
<td></td>
</tr>
<tr>
<td>insulated-backed, ½&quot;</td>
<td>1.82</td>
<td></td>
</tr>
<tr>
<td>Softwoods, fir and pine</td>
<td>1.25</td>
<td></td>
</tr>
<tr>
<td>Hardwoods, maple and oak</td>
<td>0.91</td>
<td></td>
</tr>
<tr>
<td>Plywood, ¾&quot;</td>
<td>1.25</td>
<td>0.47</td>
</tr>
<tr>
<td>Plywood, ½&quot;</td>
<td>1.25</td>
<td>0.62</td>
</tr>
<tr>
<td>Particleboard, medium density</td>
<td>1.06</td>
<td></td>
</tr>
<tr>
<td>Hardboard, tempered, ¼&quot;</td>
<td>1.00</td>
<td>0.25</td>
</tr>
<tr>
<td>Insulating sheathing, ¼&quot;</td>
<td>2.06</td>
<td></td>
</tr>
<tr>
<td>Gypsum or plasterboard, ½&quot;</td>
<td>0.45</td>
<td></td>
</tr>
<tr>
<td>Wood siding, lapped, ½&quot;×8&quot;</td>
<td>0.81</td>
<td></td>
</tr>
<tr>
<td>Asphalt shingles</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>Wood shingles</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>Windows (includes surface conditions)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single glazed</td>
<td>0.91</td>
<td></td>
</tr>
<tr>
<td>with storm windows</td>
<td>2.00</td>
<td></td>
</tr>
<tr>
<td>Insulating glass, ¼&quot; air space</td>
<td>1.69</td>
<td></td>
</tr>
<tr>
<td>double pane</td>
<td>2.56</td>
<td></td>
</tr>
<tr>
<td>triple pane</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doors (exterior, includes surface conditions)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wood, solid core, 1¾&quot;</td>
<td>3.03</td>
<td></td>
</tr>
<tr>
<td>Metal, urethane core, 1¾&quot;</td>
<td>2.50</td>
<td></td>
</tr>
<tr>
<td>Metal, polystyrene core, 1¾&quot;</td>
<td>2.13</td>
<td></td>
</tr>
<tr>
<td>Air space (¾&quot; to 4&quot;)</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>Surface conditions</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>Inside surface</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>Outside surface</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The insulation value of fiberglass varies with batt thickness. Check package label.

Ways of expressing the value of insulation are:

R = thermal resistance, hr-ft²-F/Btu.
It is the resistance to heat flow of 1 ft² of material when the temperature difference between the two sides is 1 F.
R is an additive quantity; 2" of a material has twice the R-value of 1". Also the individual R-values for all materials in a given section of a structure can be added together to obtain a total R-value.

Rₜ = total thermal resistance.
It is the total resistance of an entire wall, ceiling, etc. section, including the air film coefficients.

U = overall coefficient of heat transmission, Btu/hr-ft²-F = 1/Rₜ.
It is the heat in Btu/hr that passes through an entire wall, ceiling, etc. section of 1 ft², in one hour per 1 F temperature difference between the air on the warm side and the air on the cold side.

k = thermal conductivity, Btu-in./ft²-F-hr.
It is the heat in Btu/hr that passes through a piece of material 1" thick and 1 ft², when the temperature difference between the two sides is 1 F.

C = thermal conductance, Btu/ft²-F-hr.
C is like k, except it is given for the total thickness: k for glass wool = 0.29; C for 3" glass wool = 0.10. By convention, C does not usually include the effects of boundary layer resistances.

In the following discussion, R is used because the insulation value of a wall is easier to calculate, and many insulations are marked with their R-value.

Example 1:
Given the wall in Fig 631-7, find the total R-value.
From Table 631-1 find the R-values for each material.

<table>
<thead>
<tr>
<th>Material</th>
<th>R value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside surface</td>
<td>0.17</td>
</tr>
<tr>
<td>Wood Siding, 8&quot; lapped</td>
<td>0.81</td>
</tr>
<tr>
<td>3-j&quot; Blanket Insulation, R=11</td>
<td>11.00</td>
</tr>
<tr>
<td>Vapor Barrier</td>
<td>0.00</td>
</tr>
<tr>
<td>Plywood, ¾&quot;</td>
<td>0.62</td>
</tr>
<tr>
<td>Inside surface</td>
<td>0.68</td>
</tr>
<tr>
<td>Total R</td>
<td>13.28</td>
</tr>
</tbody>
</table>

Fig 631-7. R-value of a wall section.

By adding the individual R-values, we find the wall has a total R-value of 13.28. Note that the blanket insulation provides more than 80% of the total R. Example calculations for other wall constructions are shown in Table 631-2.
Table 631.3. Insulation R-values for other construction.

<table>
<thead>
<tr>
<th>Roof Type</th>
<th>R&lt;sub&gt;T&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal roofing, 1/4&quot; insulating sheathing</td>
<td>2.91</td>
</tr>
<tr>
<td>Metal roofing, 0.4&quot; expanded polyurethane</td>
<td>3.35</td>
</tr>
<tr>
<td>Metal roofing, 1&quot; molded polystyrene, 1 pcf</td>
<td>5.85</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ceiling Type</th>
<th>R&lt;sub&gt;T&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>2&quot; expanded polyurethane</td>
<td>13.35</td>
</tr>
<tr>
<td>1/4&quot; plywood, 4&quot; glass or mineral wool fill insulation</td>
<td>13.47</td>
</tr>
<tr>
<td>Metal roofing, R-19 blanket insulation</td>
<td>19.85</td>
</tr>
<tr>
<td>1/2&quot; plywood, 8&quot; glass or mineral wool fill insulation</td>
<td>25.47</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Door Type</th>
<th>R&lt;sub&gt;T&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4&quot; plywood, R-2 blanket insulation, 1/4&quot; air space, 1/4&quot; plywood</td>
<td>4.99</td>
</tr>
<tr>
<td>1/4&quot; plywood, 1&quot; polystyrene molded, 1 pcf</td>
<td>7.99</td>
</tr>
<tr>
<td>1/2&quot; air space, 1/2&quot; plywood</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Floor perimeter (per foot of exterior wall)</th>
<th>R&lt;sub&gt;T&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>1.23</td>
</tr>
<tr>
<td>Concrete, with 2&quot;x24&quot; of rigid insulation around perimeter</td>
<td>2.22</td>
</tr>
</tbody>
</table>

Residences

Unlike some farm buildings, homes must be insulated. The amount of insulation for a home depends primarily on economics and climate. Consider insulation as an investment against reduced heating and cooling bills. Often in new construction, installing the proper amount of insulation also reduces the size of the heating and cooling system.

Recommended insulation levels are often based on the heating season’s degree day total. "Winter degree days" estimates winter season severity by comparing weather data with 65 F. A large number indicates average temperatures farther below 65 F and/or a longer season. The number of degree days for each day is calculated as the difference between 65 F and the day’s average temperature. The number of degree days during the heating season is the total of the daily differences for the season. See Fig 631-8 for the number of degree days for your area. To qualify for financing by the Farmers Home Administration (FmHA), new residences must meet the minimum requirements in Table 631-4. Note that these R-values are minimums.

![Fig 631-8. Winter degree days.](image)

Accumulated difference between 65 F and average daily temperature for all days in the heating season.

Table 631.4. Home insulation levels.

<table>
<thead>
<tr>
<th>Winter degree days</th>
<th>Ceiling</th>
<th>Walls</th>
<th>Floors*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 or less</td>
<td>20</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>1001 to 2500</td>
<td>25</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>2501 to 6000</td>
<td>33</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>6001 or more</td>
<td>38</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

*For floors of heated spaces over unheated basements, unheated garages or unheated crawl spaces.

![Fig 631-9. Example of residence insulation.](image)

R<sub>Wall</sub> = 21.12
R<sub>Ceiling</sub> = 39.41

Farm buildings

The amount of insulation needed in farm buildings depends on many factors, such as the expected outside temperature (degree days), number and size of animals housed, desired inside temperature, and economics.

"Cold" buildings have indoor conditions about the same as outside conditions. Examples are machinery storages, cold free stall barns, and open-front livestock buildings. Minimum insulation is frequently recommended in the roof of these buildings to reduce solar heat gain in summer and condensation in winter.

Modified environment buildings rely on animal heat and controlled, natural ventilation to remove moisture and maintain the desired inside temperature. Examples are warm free stall barns, poultry production buildings, and swine finishing units.

Supplementally heated buildings require extra heat to maintain the desired inside temperature. Examples include farrowing buildings, some calf buildings, farm shops, and offices. Cold and modified buildings requiring supplemental heat in a small area, such as brooders in an open-front building, are not classified as supplementally heated.